Statistical Applications Using Minitab

May 14, 2014

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Los Angeles Section ASQ

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The Statistician

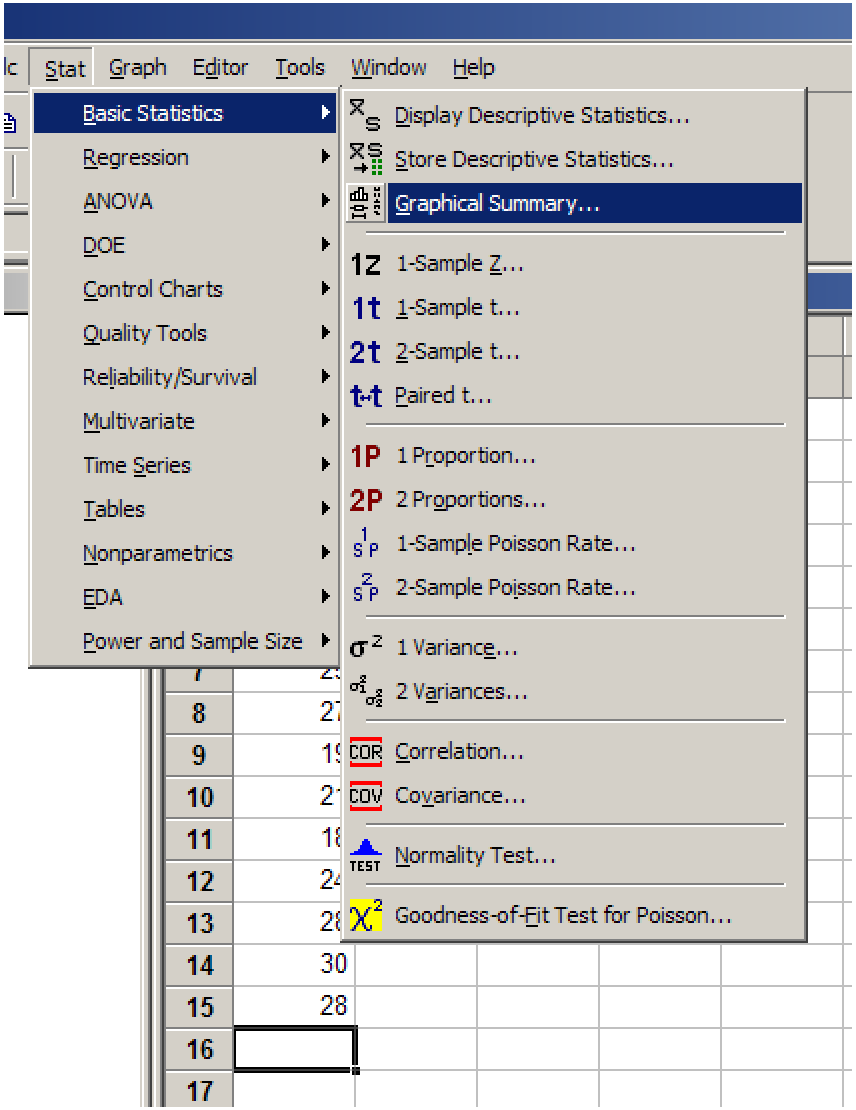
The Statistician is a person who poses as an exacting expert on the basis of being able to turn out with prolific fortitude infinite strings of incomprehensible formulae calculated with micrometric precision from vague assumptions which are based upon debatable figures taken from inconclusive experiments carried out with instruments of problematical accuracy by persons of doubtful reliability and questionable mentality.

Key Take-Aways

- Minitab is a Simple and Powerful Tool in Data Analysis and Display
- Always try to Graphically Display your data
  - Easier to understand
  - Draws picture of what is really happening
  - Check for Patterns
    - Is it Normal?
    - Time Series for trends, changes, effects
- Organize and Plan you Data Collection first
- Analyze the results and recheck any assumptions
- Have Fun. Software has made data analysis easier, faster, and funner!

A Warm-up Look

- First record 10-15 data points in the first column.
- Let's name the column "Results"
- Go to Stat>Basic Statistics>Graphical Summary



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How Do I Look At The Data ?

- It's PGA, Baby!!
  - Practical (Are there any obvious problems or patterns?) ANOB or ANOG
  - Graphical (Look at the Pictures)
    - Two different situations, two histograms requiring different approaches
  - Analytical (Check out the Statistics)

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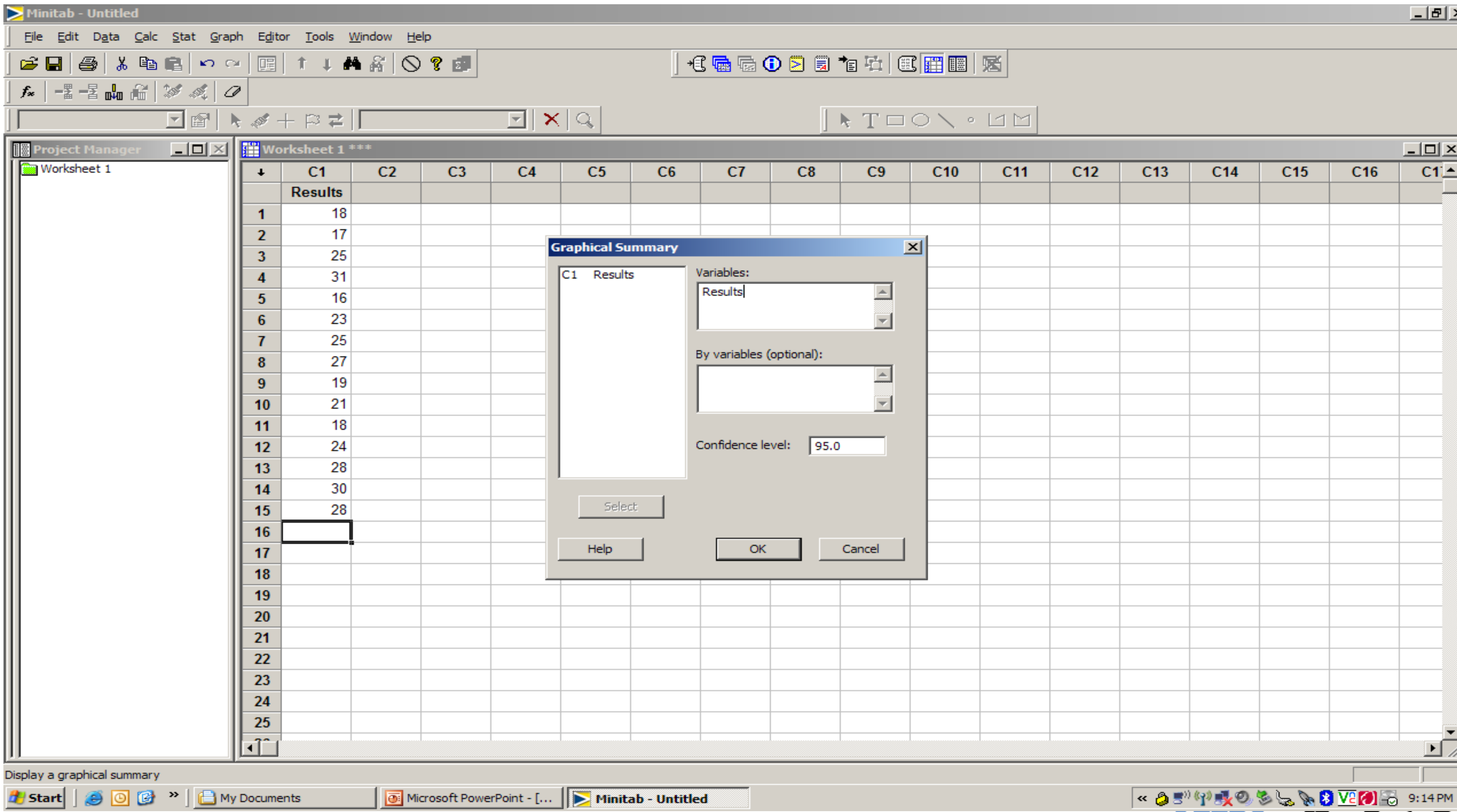
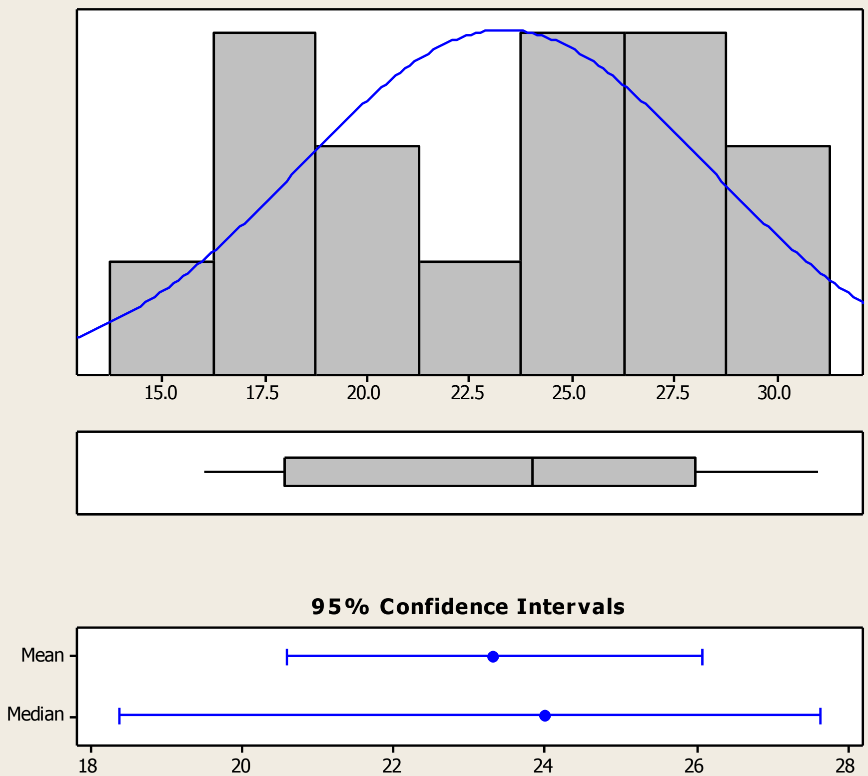
You should end up with something like this



ality

And a Graphical Summary

Summary for Results



Anderson-Darling Normality Test	
A-Squared	0.33
P-Value	0.470
Mean	23.333
StDev	4.938
Variance	24.381
Skewness	-0.02525
Kurtosis	-1.35184
N	15
Minimum	16.000
1st Quartile	18.000
Median	24.000
3rd Quartile	28.000
Maximum	31.000
95% Confidence Interval for Mean	
20.599	26.068
95% Confidence Interval for Median	
18.374	27.626
95% Confidence Interval for StDev	
3.615	7.787

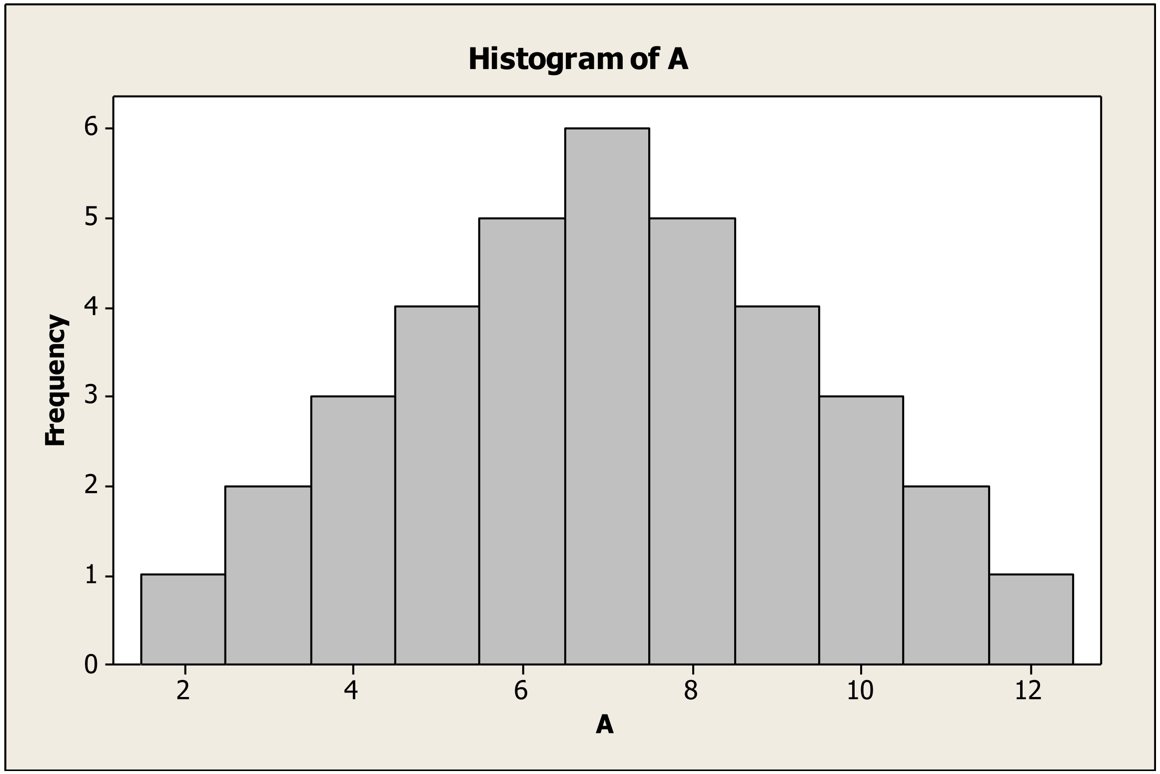
More About Displaying Data

A histogram is a graphical display of tabulated frequencies, which are shown as bars. It illustrates what portion of cases fall into each of several categories.

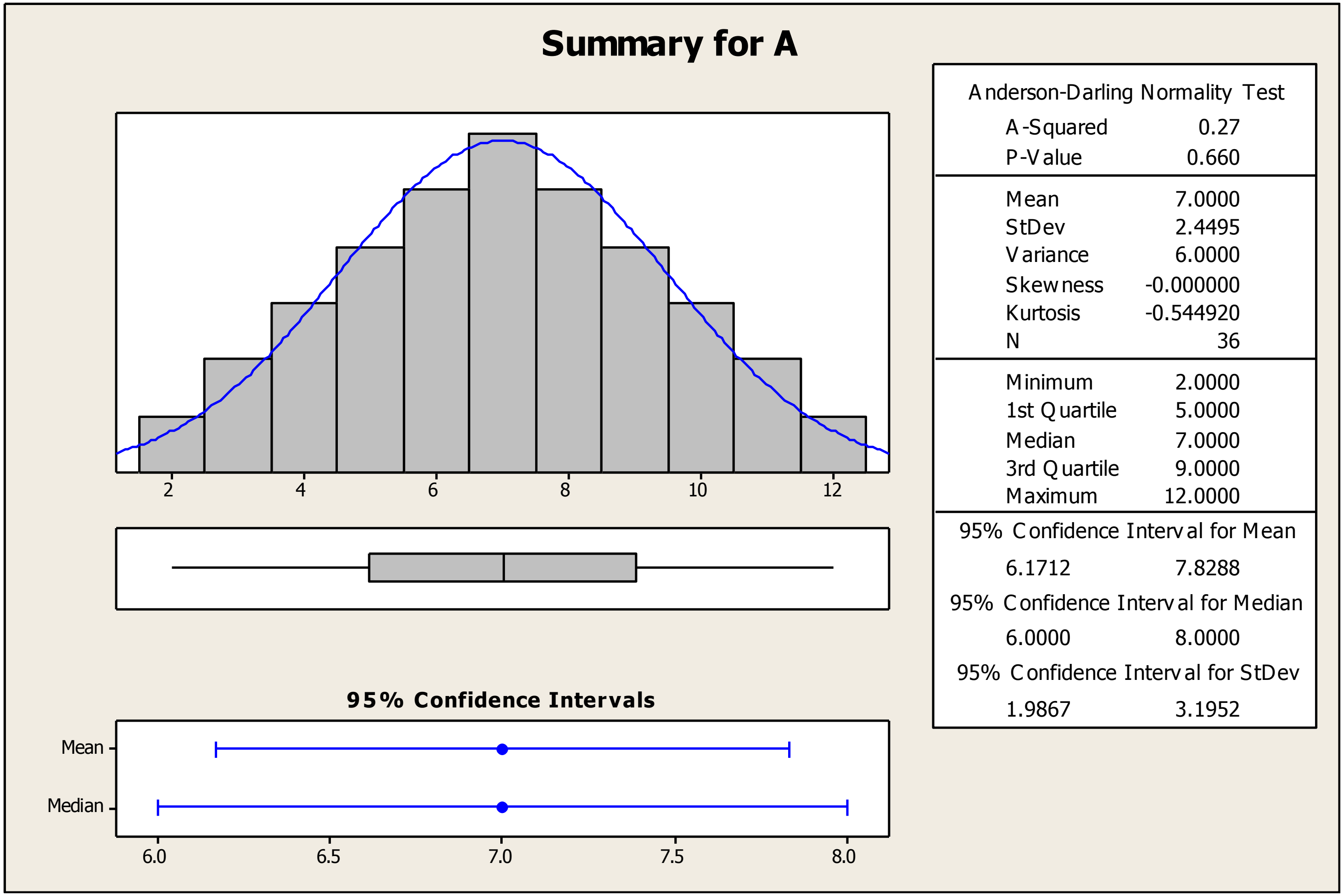
Lets look at an example for the following data:

4,9,7,2,11,7,12,6,8,5,3,8,7,6,9,4,9,6,5,10,
8,5,7,3,8,6,10,9,10,7,11,6,8,5,4,7.

The histogram would look like this:



A Quantitative Look At The Data



How about this data: 4,9,7,5,10,7,11,8,8,5,3,8,9,5,9,3,5,9,5,10

6,7,9,4,8,2,10,8,10,6,8,6,8,5,4,11.

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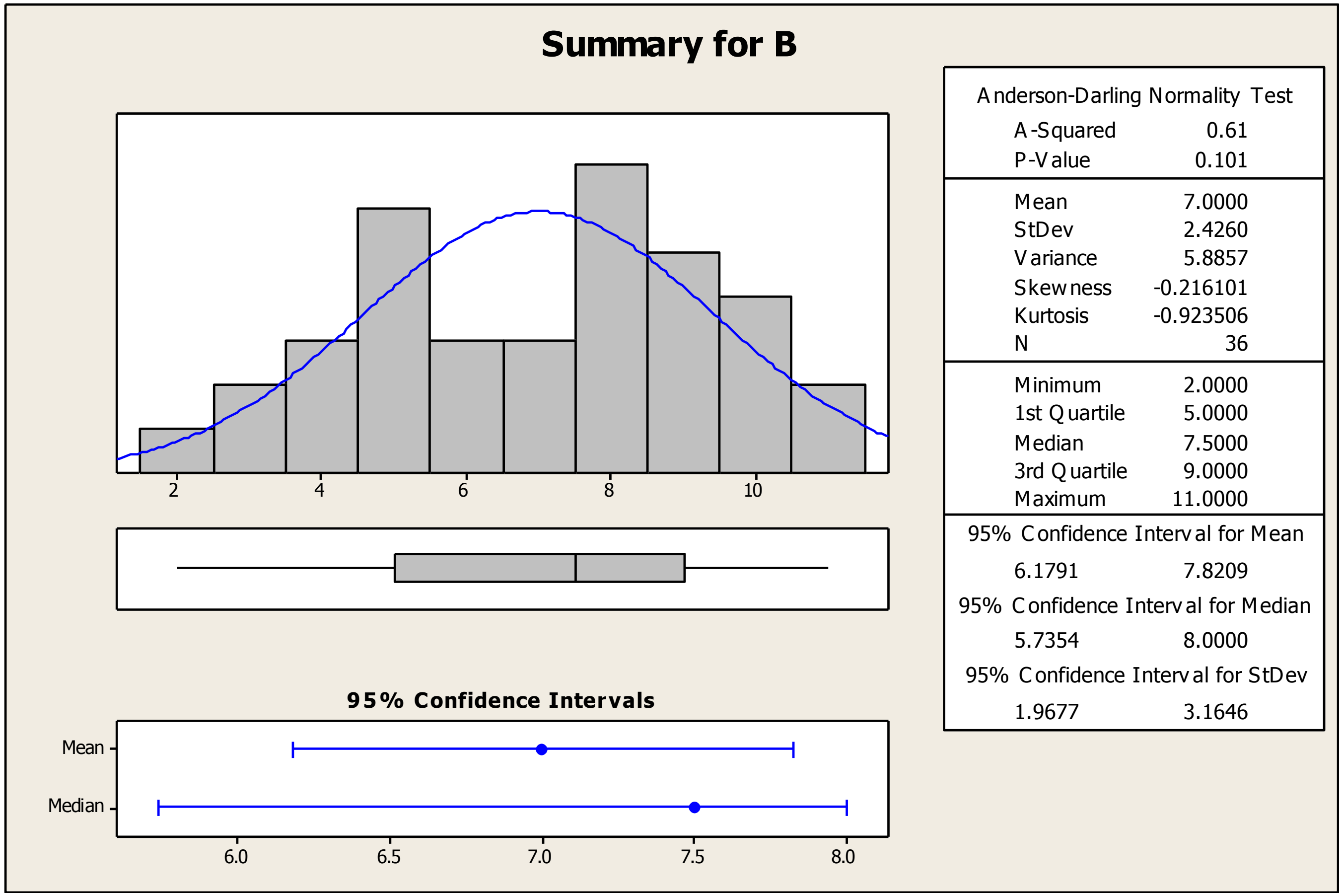
How Does The New Data Set Look?



The Mean is the same and the Std Dev is almost identical, but the shape of their distribution is very different. What type of distribution is this?

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Distributions

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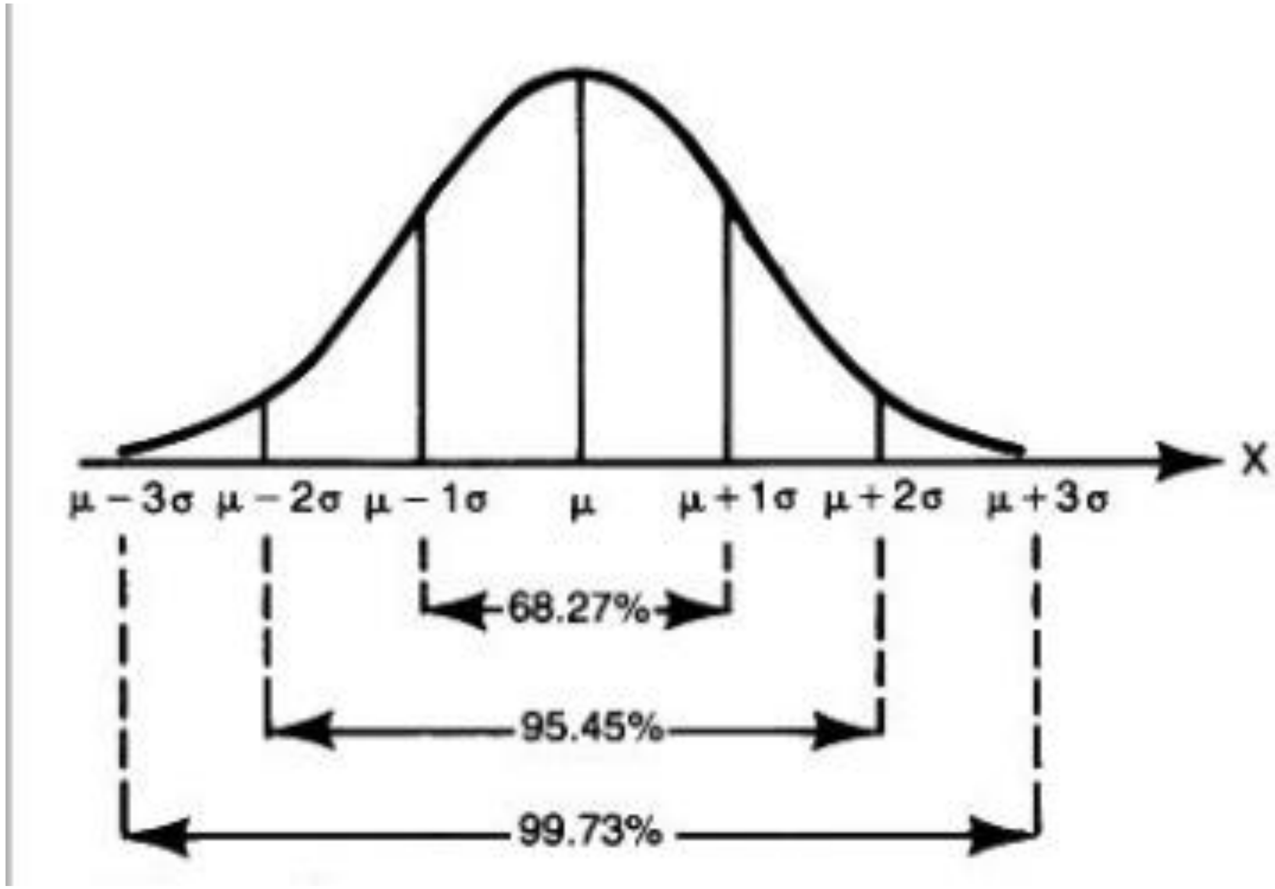
The Normal Distribution
aka The Bell Curve



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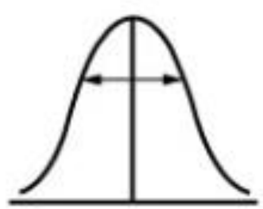
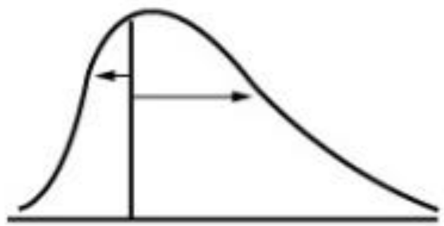
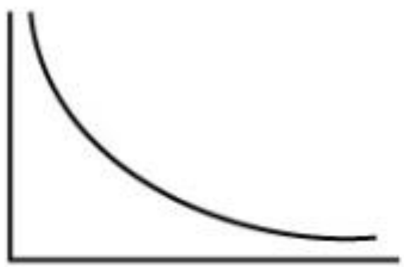
Non-Normal Distributions



A. Normal Distribution

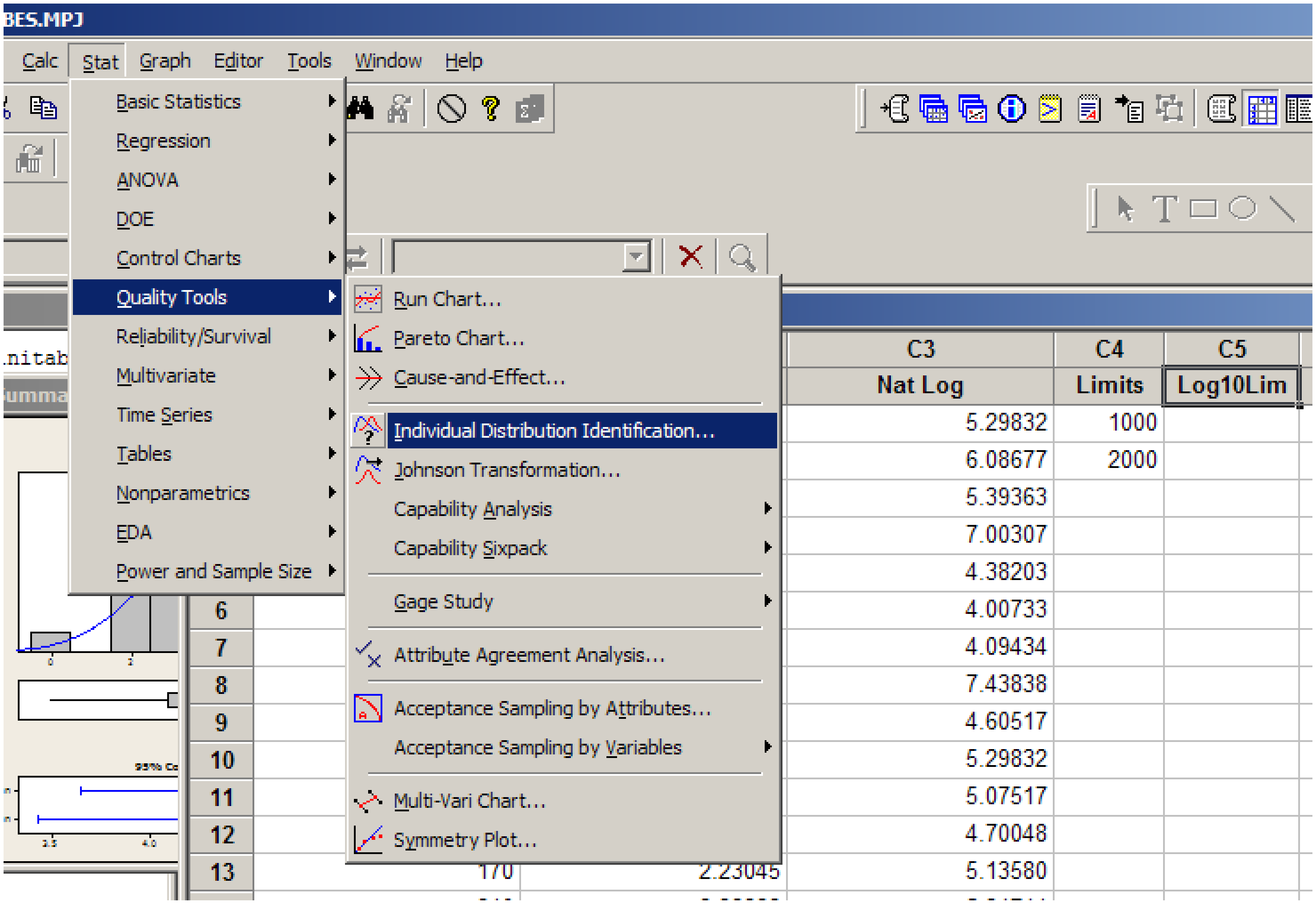


B. Skewed Distribution

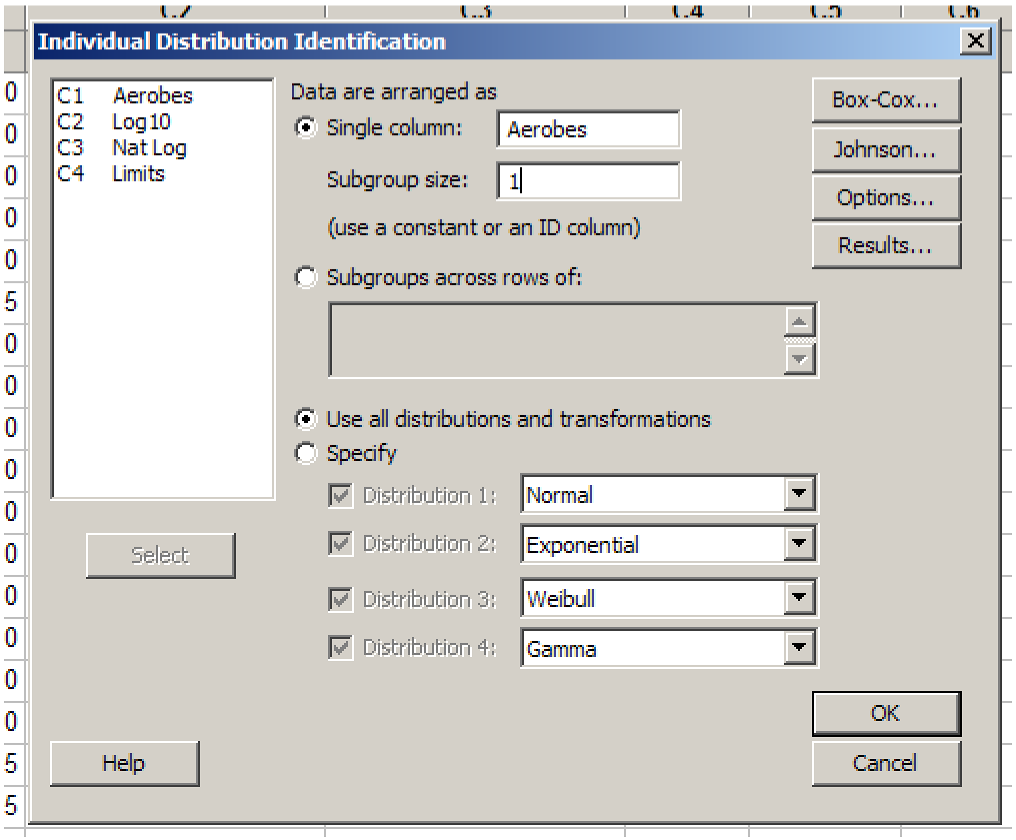


C. Exponential Distribution

Checking Many Distributions At The Same Time



Individual Distribution Identification



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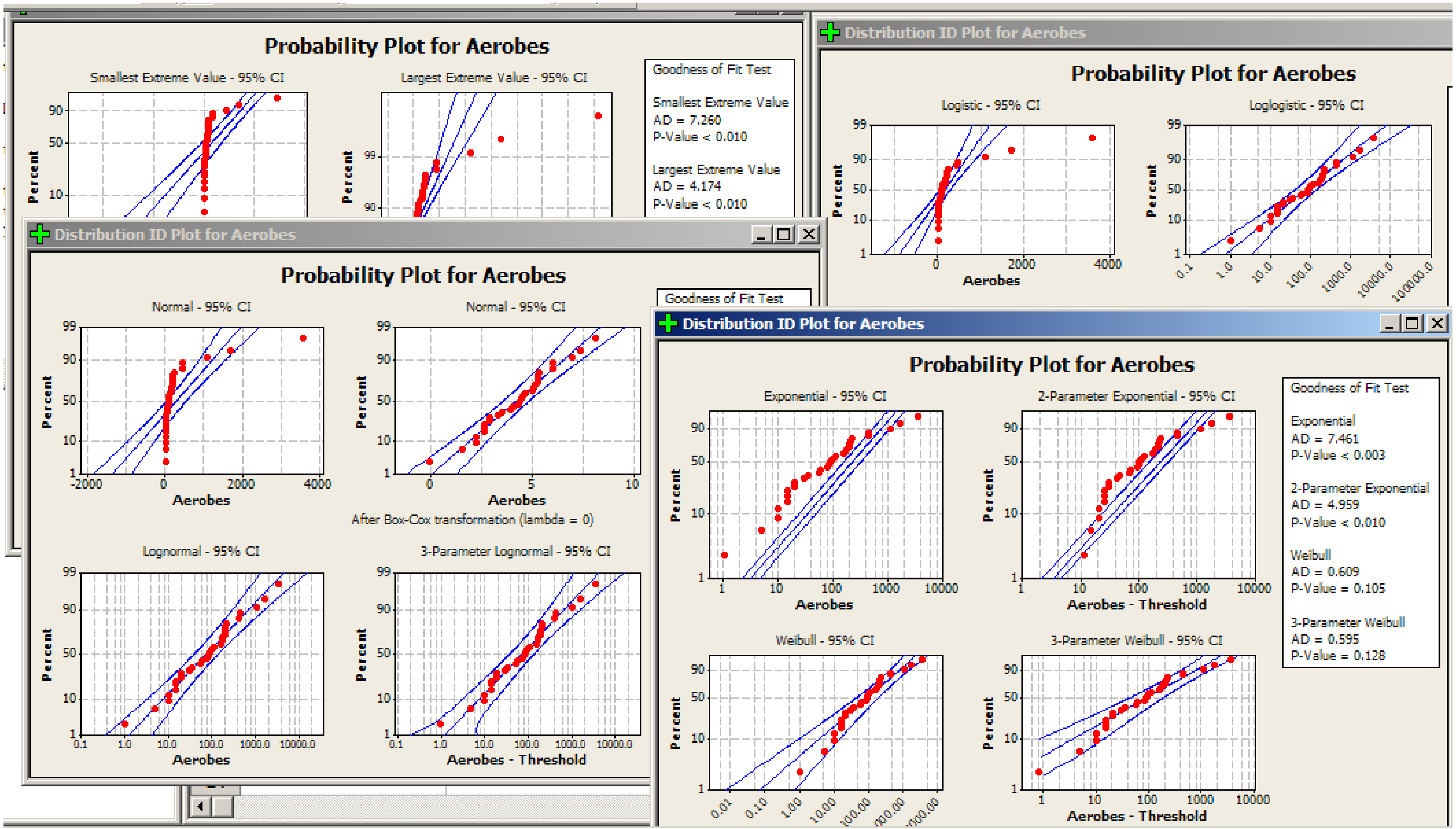
Here Is What You Get



Holy Crap !

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Transformation

Your Data Analysis Choices

- Normal Distribution (Bell Curve)
- Transform to Normalize the Data
- Utilize Non-parametric Statistics
- Treat as Discrete (Attribute Data)

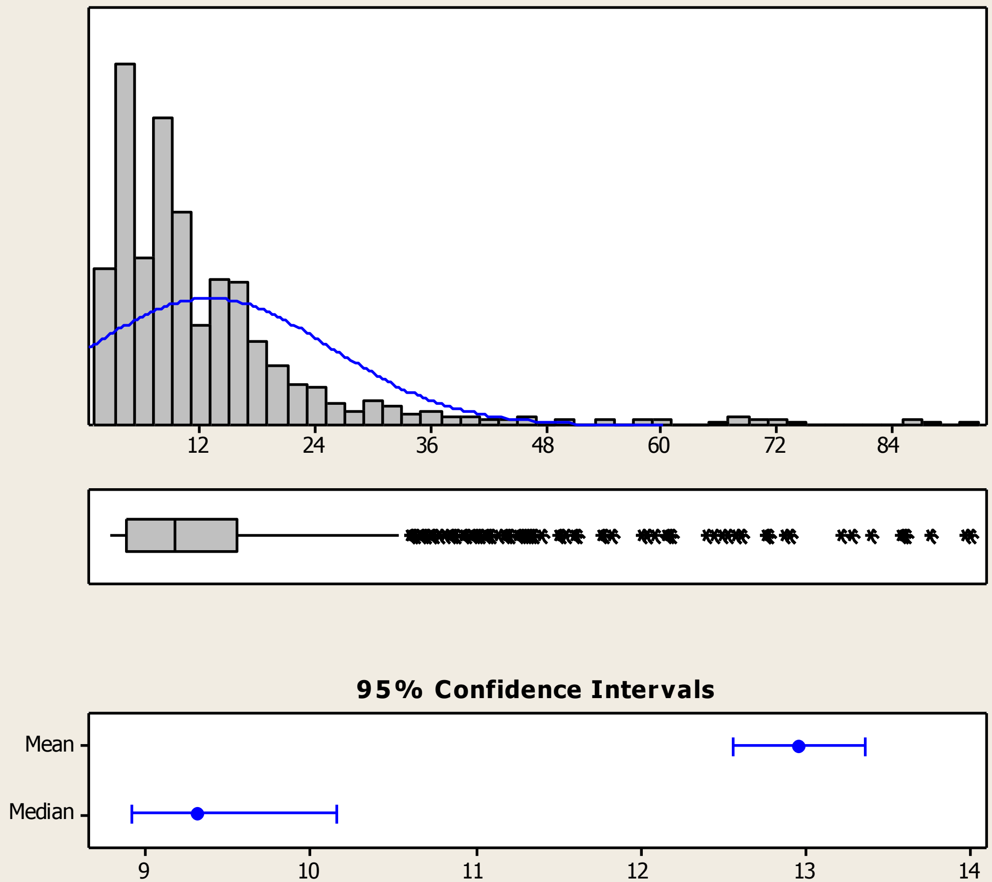
Transformations

Make a transformation of the original characteristic to a new characteristic that is normally distributed. These transformations are useful for (a) achieving normality of measured results, (b) satisfying the assumption of equal sample variances required in certain tests, and (c) satisfying the assumption of additivity of effects in certain tests.

From Joseph Juran, Quality Control Handbook

Some Things Just Aren't Normal

Summary for July - Ground



Anderson-Darling Normality Test	
A-Squared	285.69
P-Value <	0.005
Mean	12.956
StDev	12.674
Variance	160.621
Skewness	2.9189
Kurtosis	11.0296
N	3921
Minimum	2.650
1st Quartile	4.210
Median	9.320
3rd Quartile	15.840
Maximum	92.020
95% Confidence Interval for Mean	
12.559	13.353
95% Confidence Interval for Median	
8.920	10.160
95% Confidence Interval for StDev	
12.399	12.961

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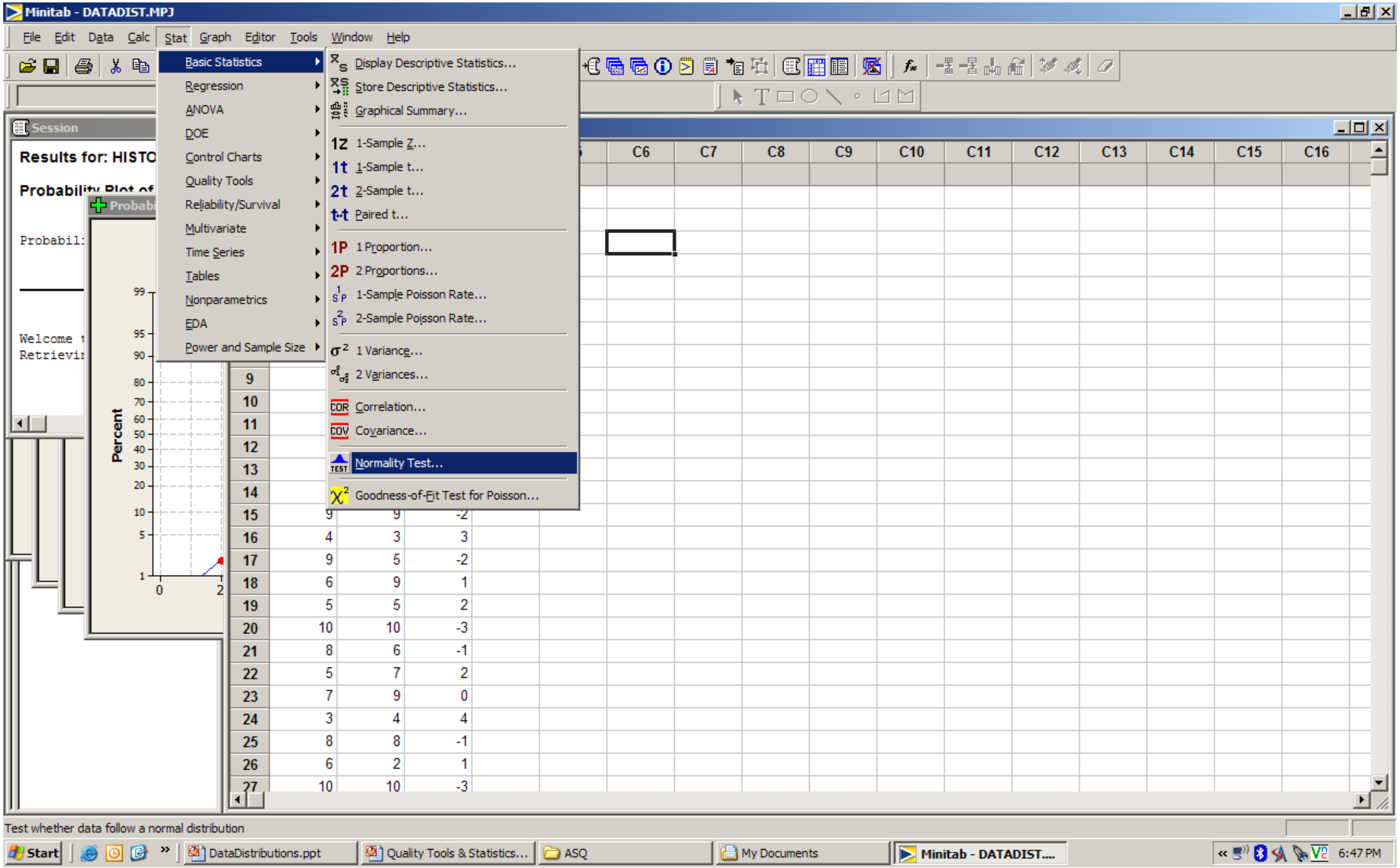
Testing For Normality



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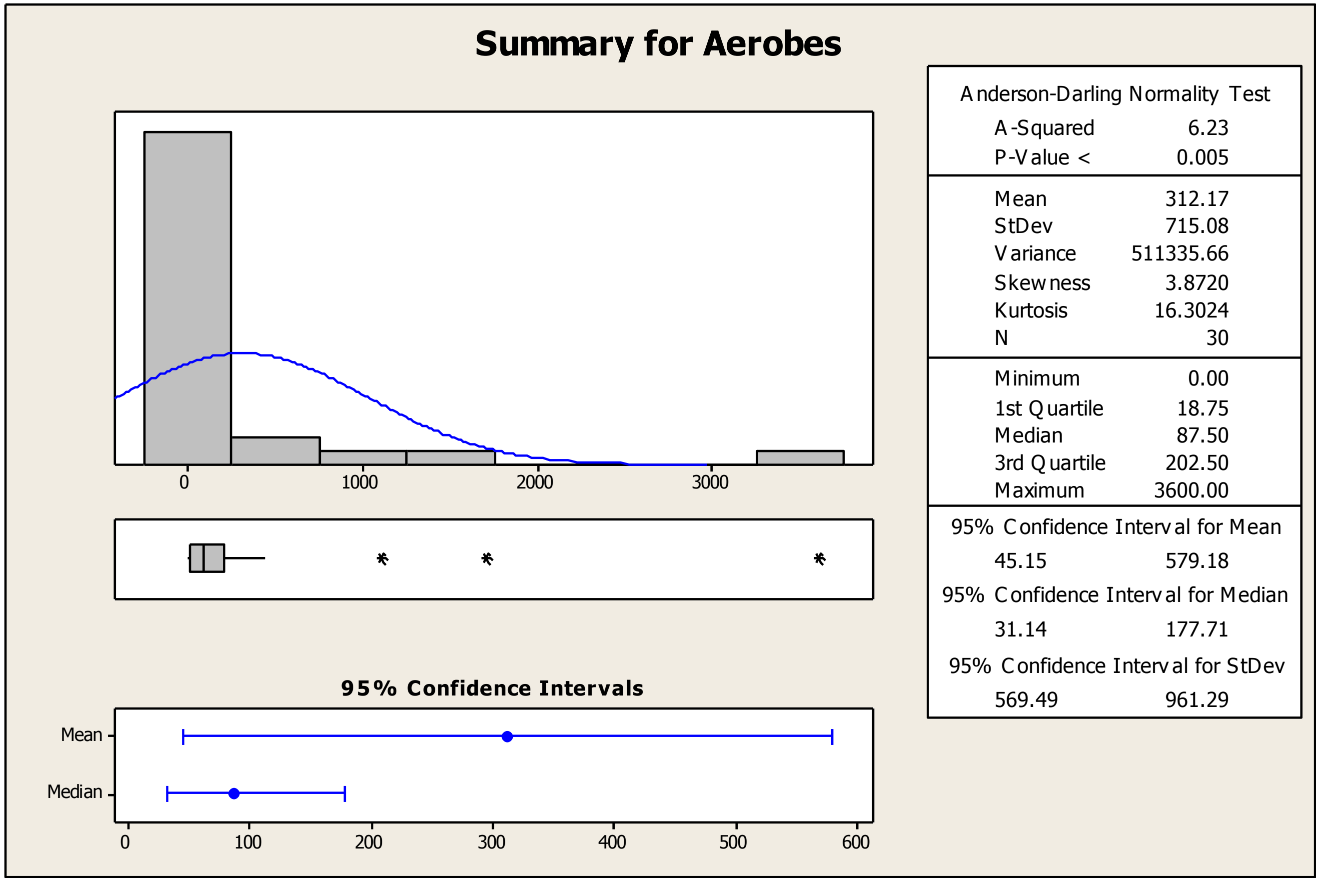
A Study - Aerobes

<u>Aerobes</u>
200
440
220
1100
80
55
60
1700
100
200
160
110
170
210
3600
90
85
35
430
180
5
15
1
30
10
10
20
15
20
15



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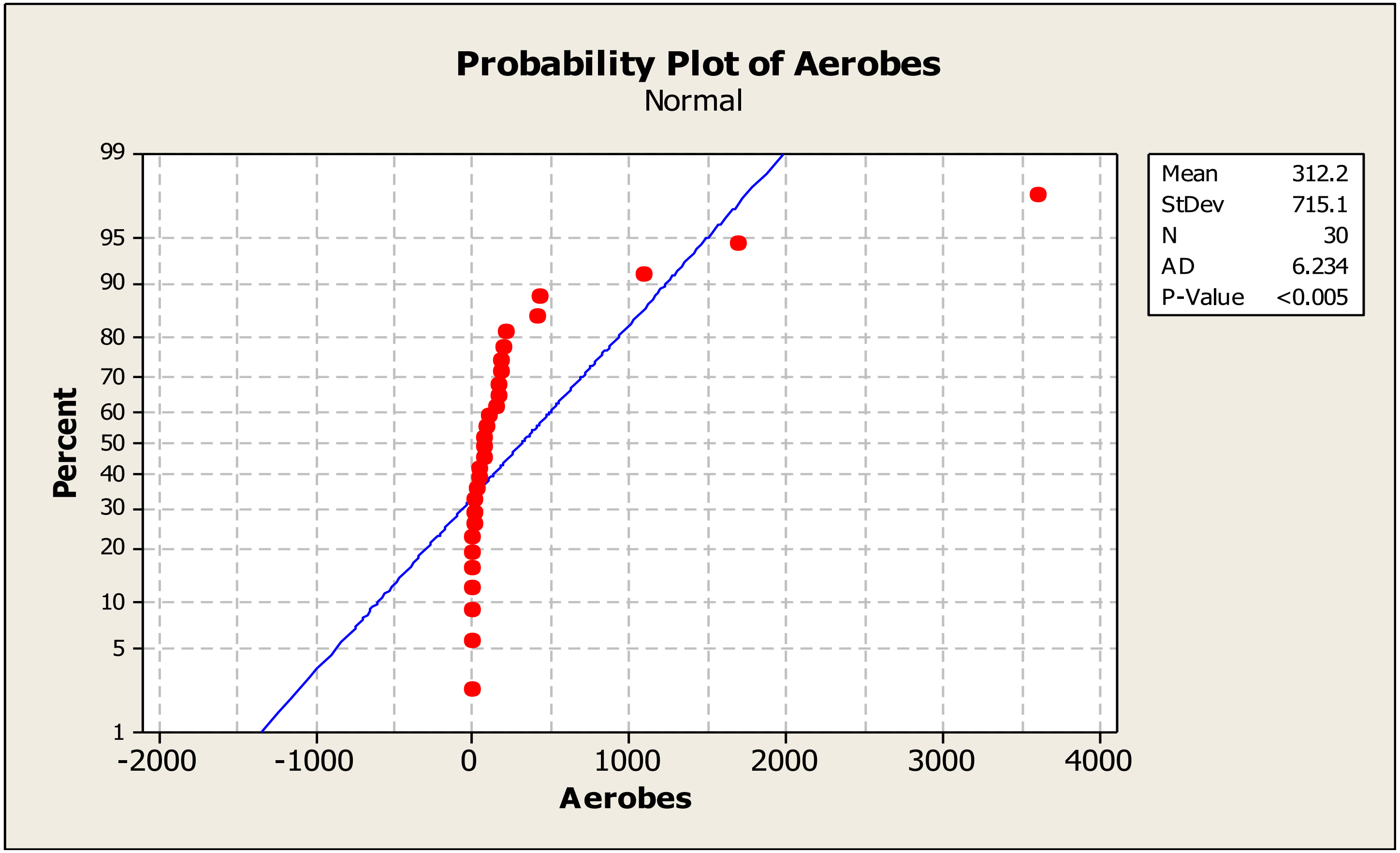
Example of Need for Transformation



What the heck happened here?

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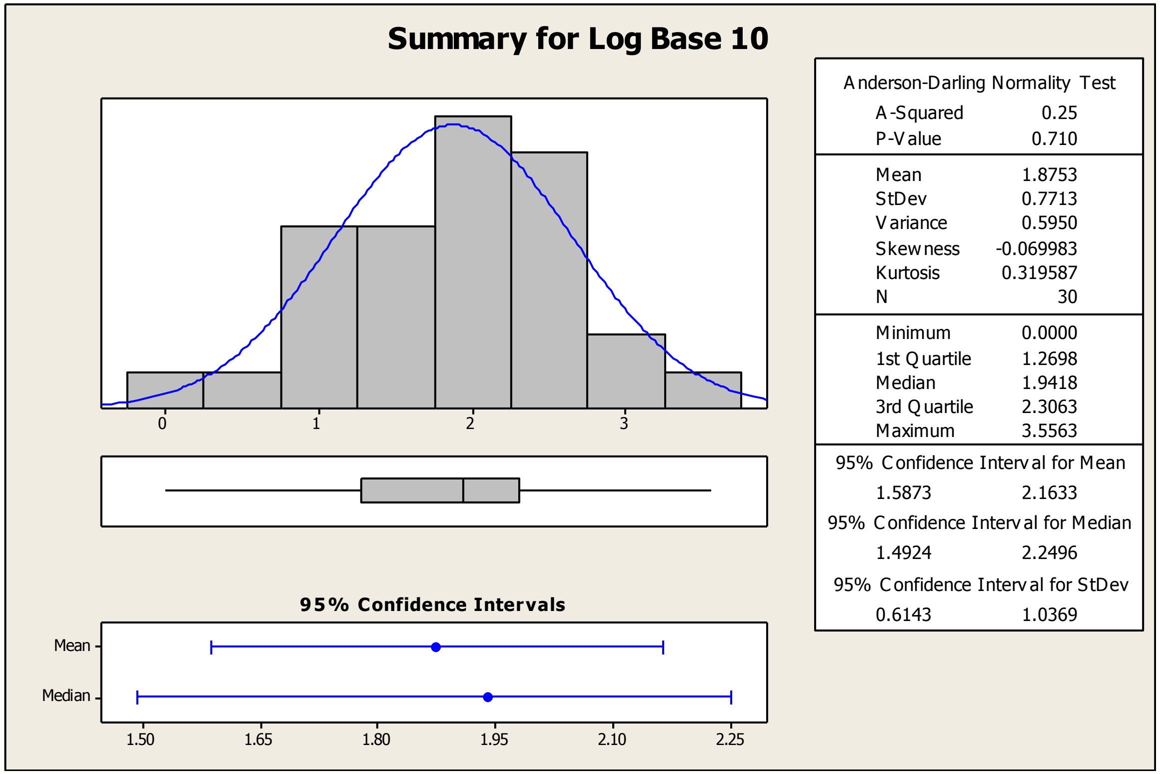
Transformation to Log Values

	Aerobes	Log10	Nat Log
1.	200	2.30103	5.298317
2.	440	2.643453	6.086775
3.	220	2.342423	5.393628
4.	1100	3.041393	7.003065
5.	80	1.90309	4.382027
6.	55	1.740363	4.007333
7.	60	1.778151	4.094345
8.	1700	3.230449	7.438384
9.	100	2	4.60517
10.	200	2.30103	5.298317
11.	160	2.20412	5.075174
12.	110	2.041393	4.70048
13.	170	2.230449	5.135798
14.	210	2.322219	5.347108
15.	3600	3.556303	8.188689
16.	90	1.954243	4.49981
17.	85	1.929419	4.442651
18.	35	1.544068	3.555348
19.	430	2.633468	6.063785
20.	180	2.255273	5.192957
21.	5	0.69897	1.609438
22.	15	1.176091	2.70805
23.	1	0	0
24.	30	1.477121	3.401197
25.	10	1	2.302585
26.	10	1	2.302585
27.	20	1.30103	2.995732
28.	15	1.176091	2.70805
29.	20	1.30103	2.995732
30.	15	1.176091	2.70805



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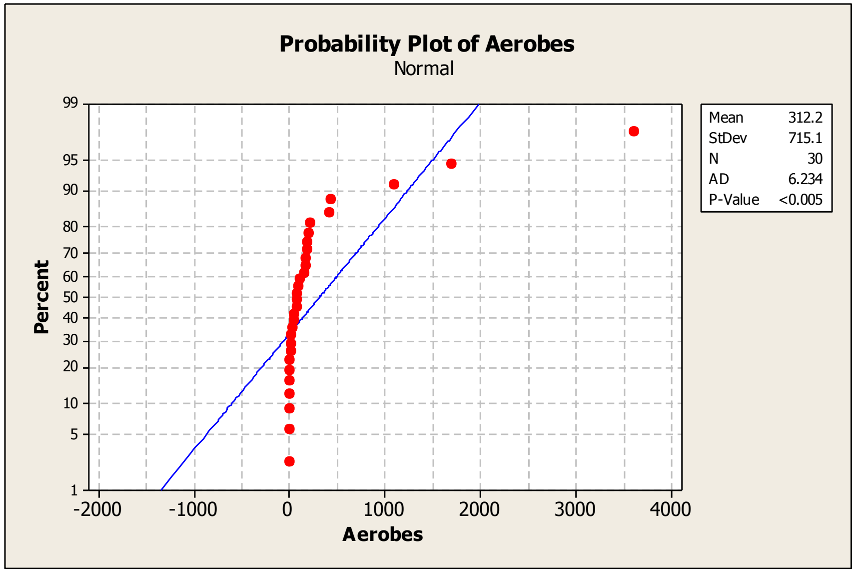
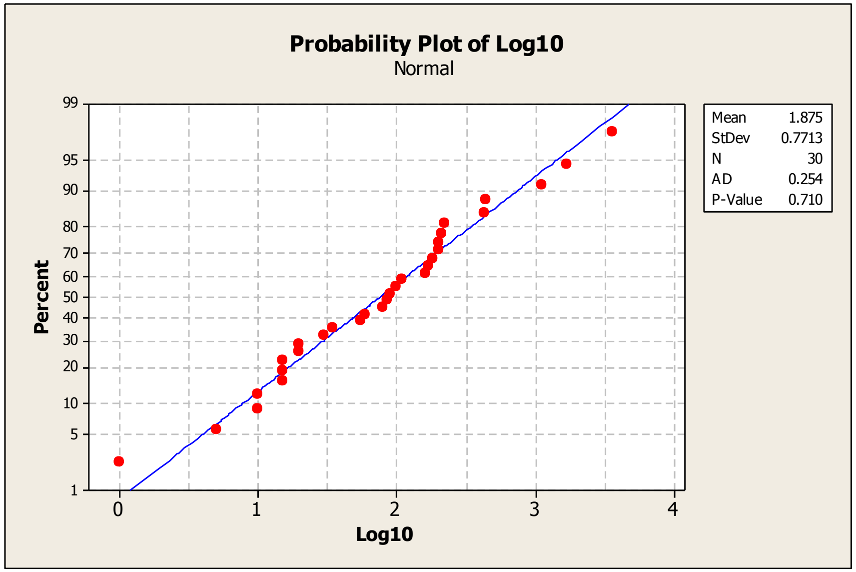
Normality Test of a Transformation





Before

After

So how did we know to take the Log value in order to normalize our data?

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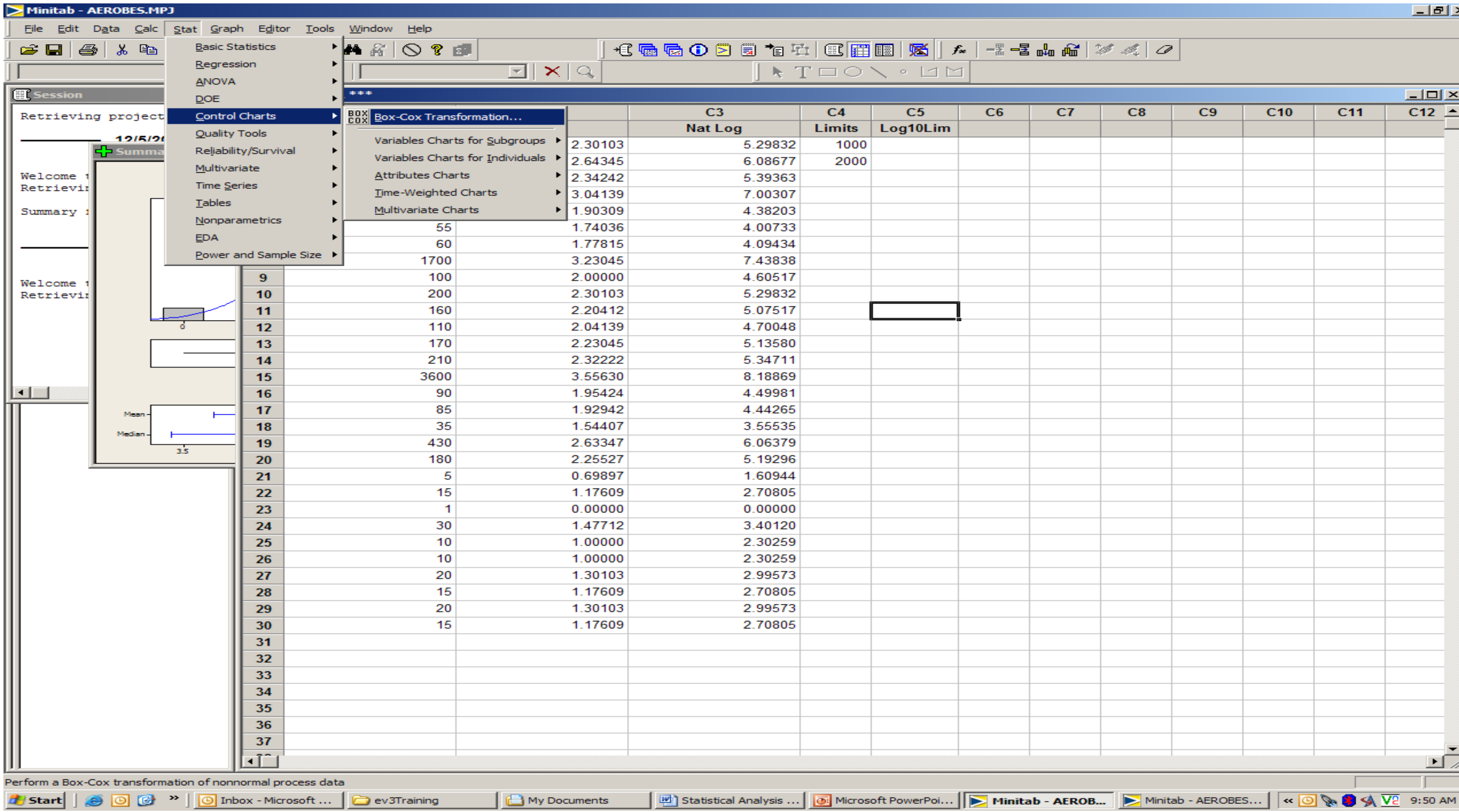
Minitab Application of Box Cox Transformation



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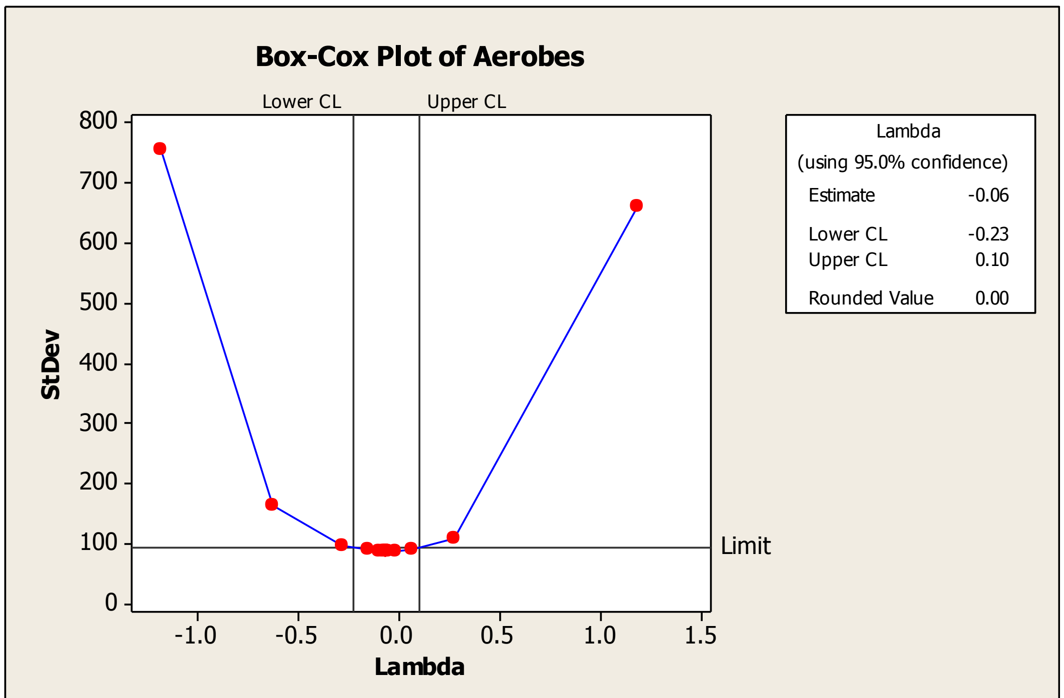
Box Cox Transformation

A transformation was considered in order to normalize the data and produce a more accurate and predictable distribution pattern. The Box-Cox Plot of the data was generated to assist in the transformation process. The plot was produced for the data as follows:

λ (Power)	Y^{λ}	Common Names
-2	$\frac{1}{Y^2}$	Reciprocal (inverse) squared
-1	$\frac{1}{Y}$	Reciprocal (inverse)
-0.5	$\frac{1}{\sqrt{Y}}$	Reciprocal square root (inverse)
0	Ln(Y)	Log
0.5	$\sqrt{Y}$	Square root
1	No transformation	---
2	Y^2	Squared



The Rounded Value for this transformation was calculated at 0.00 thus suggesting a Log (Ln(Y)) Transformation per the Box-Cox Methodology. This again was not atypical since microbial data frequently exhibits itself in an exponential manner. Both a Natural Log and a Log Base 10 transformation were calculated on the original data set using the calculator function in Minitab 15.

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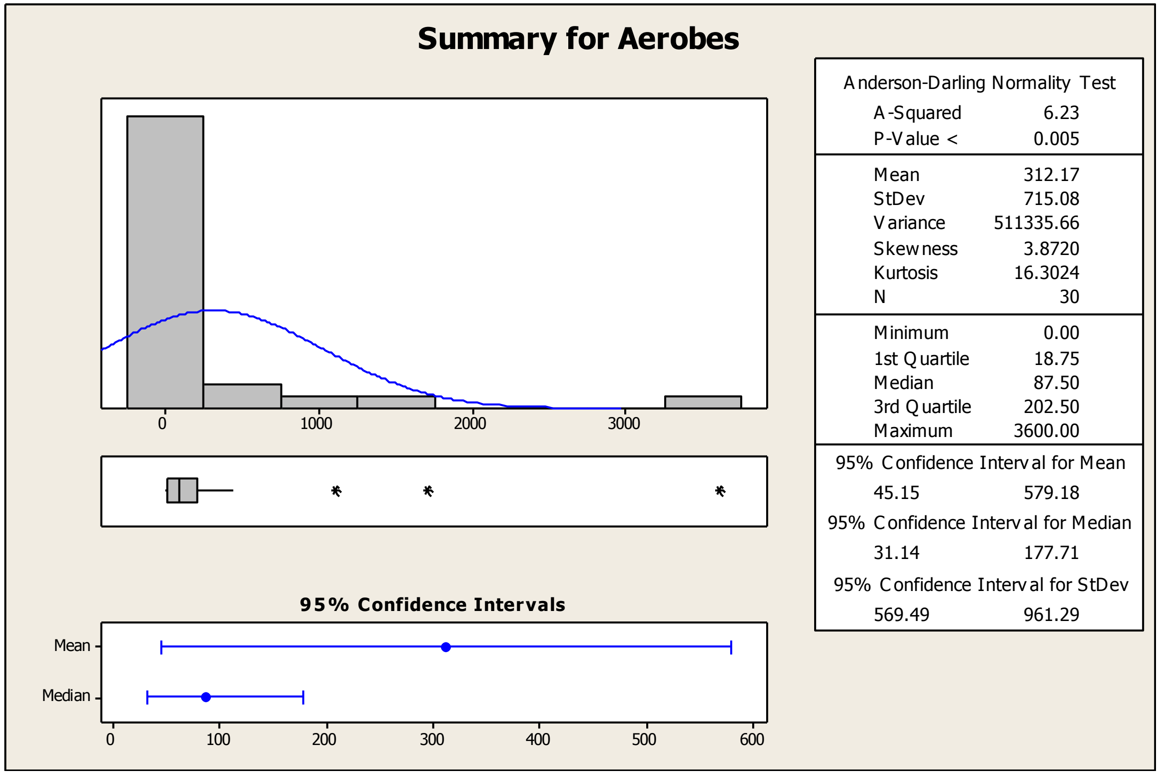
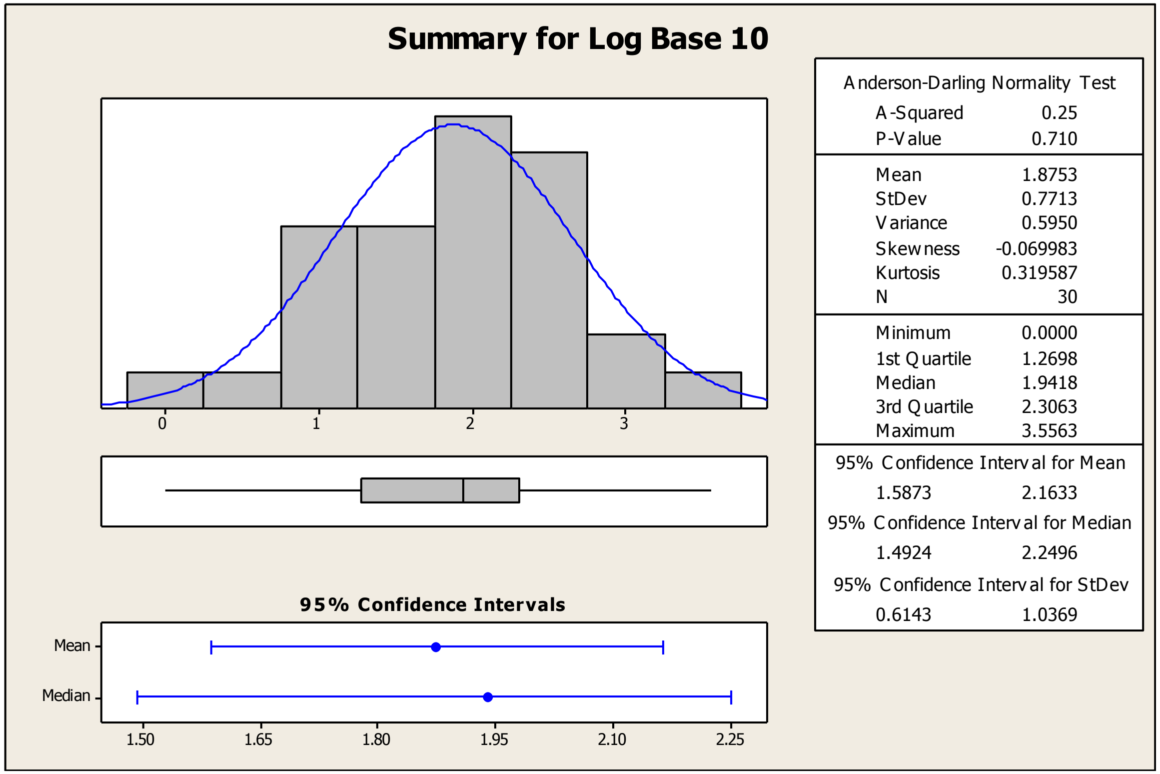
Data set – 200, 440, 220, 1100, 80, 55, 60, 1700, 100, 200, 160, 110, 170, 210, 3600, 90, 85, 35, 430, 180, 5, 15, 1, 30, 10, 10, 20, 15, 20, 15.





Conclusion

The sample with a reading of 3600 is expected to occur approximately 1 out of every 67 readings on an average and should not be considered extreme based on the distribution of the data. The sample with this count of Aerobes should be identified through a gram stain to determine the nature of its source, but unless abnormal concerns are raised based on the type of microorganism, no additional action should be taken.

And if you can't do a valid transformation

Common Nonparametric Tests

NONPARAMETRIC	FUNCTION	PARAMETRIC
1-Sample Sign*	Test the median for **1 sample** Test difference in **dependent** samples	**t test or z test**
1-Sample Wilcoxon*	Test the median difference in **dependent** samples	**t test or z test**
Mann-Whitney*	Test the median difference in 2 **independent** samples	**t test or z test**
Chi-Square/ Spearman's Rank	Test **relationships**	**Correlation / Regression**
Kruskal-Wallis/ Mood's Median*	Test the median difference in **many independent** samples	**ANOVA**
Siegel-Tukey (not in Minitab)	Test differences in spread of 2 **independent** samples	**F test**
Friedman	Test if **two factors** are significant	**2 – way ANOVA**
Runs Test	Test for **Randomness**	**No equivalent**

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Outliers

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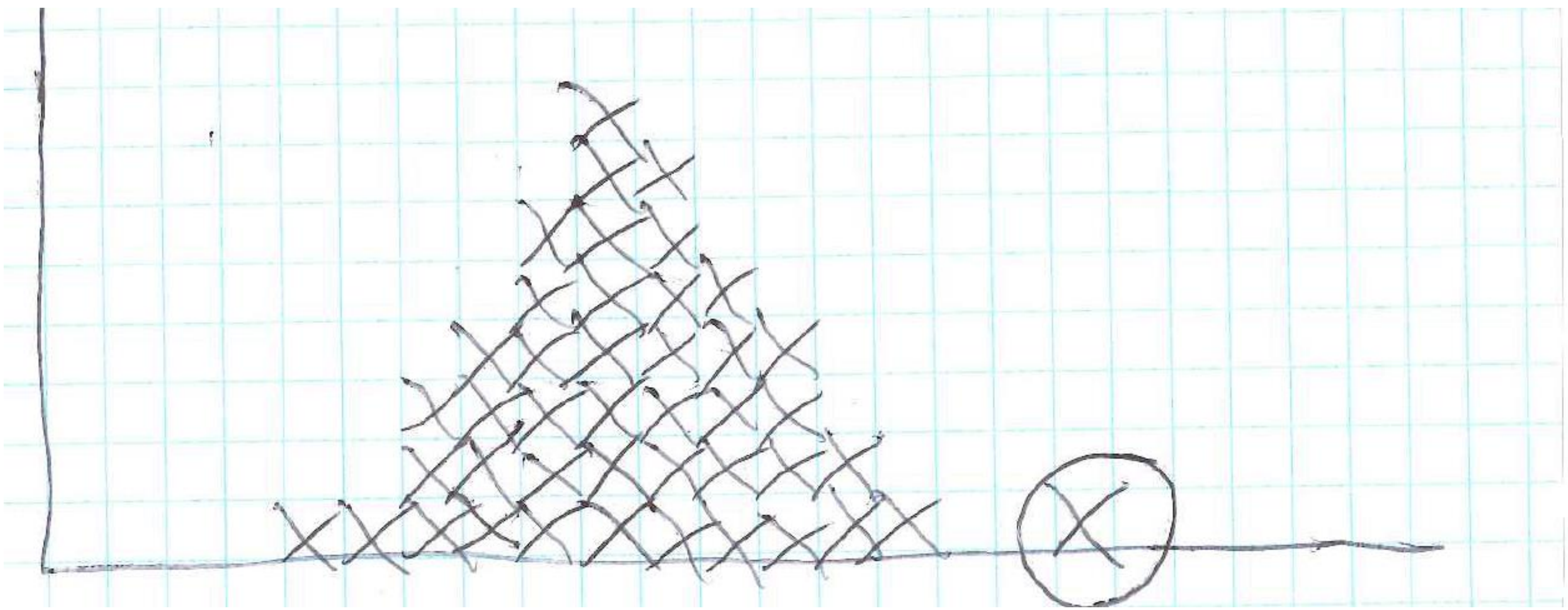


The “Oh Crap” data point aka the Outlier

- An outlier is an observation point that is distant from other observations.
- An outlier may be due to variability in the measurement or it may indicate experimental error: the latter are sometimes excluded from the data set.
- An outlier may be caused by a defective unit or a problem in the process.
- ISO 16269 defines it as “member of a small subset of observations that appears to be inconsistent with the remainder of a given sample."

- There is no rigid mathematical definition of what constitutes an outlier; determining whether or not an observation is an outlier is ultimately a subjective exercise.

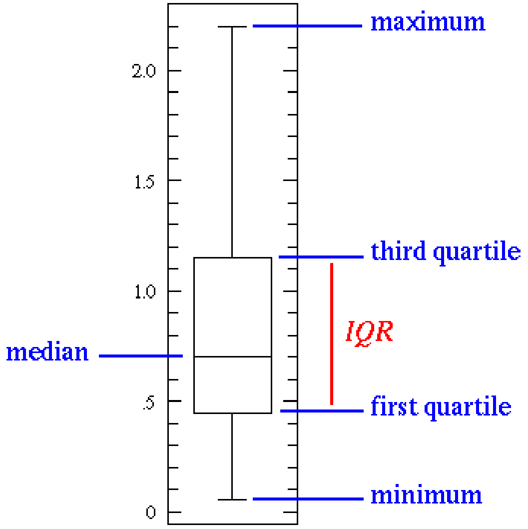
- Every effort must be made to determine what is causing the outlier to exist. Testing, equipment, operator error, materials, etc. all must be reviewed and assessment made.

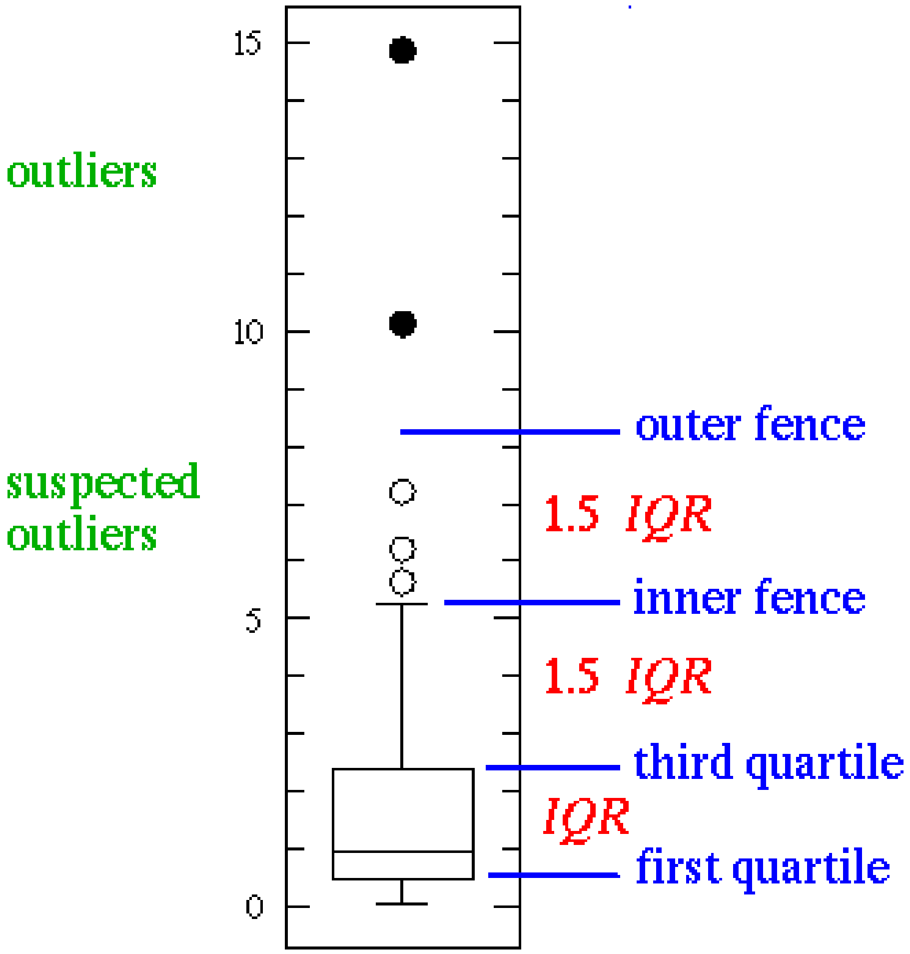
- ASTM E178-08 “Standard Practice for Dealing with Outlying Observations”
- ISO 16269-4:2010 “Statistical interpretation of data – Part 4: Detection and treatment of outliers”

- Model-based methods which are commonly used for identification assume that the data are from a normal distribution, and identify observations which are deemed “unlikely” based on mean and standard deviation:
  - Chauvenet’s criterion
  - Grubbs’ test for outliers
  - Peircece’s criterion

- Other methods flag observations based on measures such as the interquartile range. For example, if Q_1 and Q_3 are the lowest and upper quartiles respectively, then one could define an outlier to be any observation outside the range:

$$[Q_1 - k(Q_3 - Q_1), Q_3 + k(Q_3 - Q_1)]$$





http://www.physics.csbsju.edu/stats/box2.html

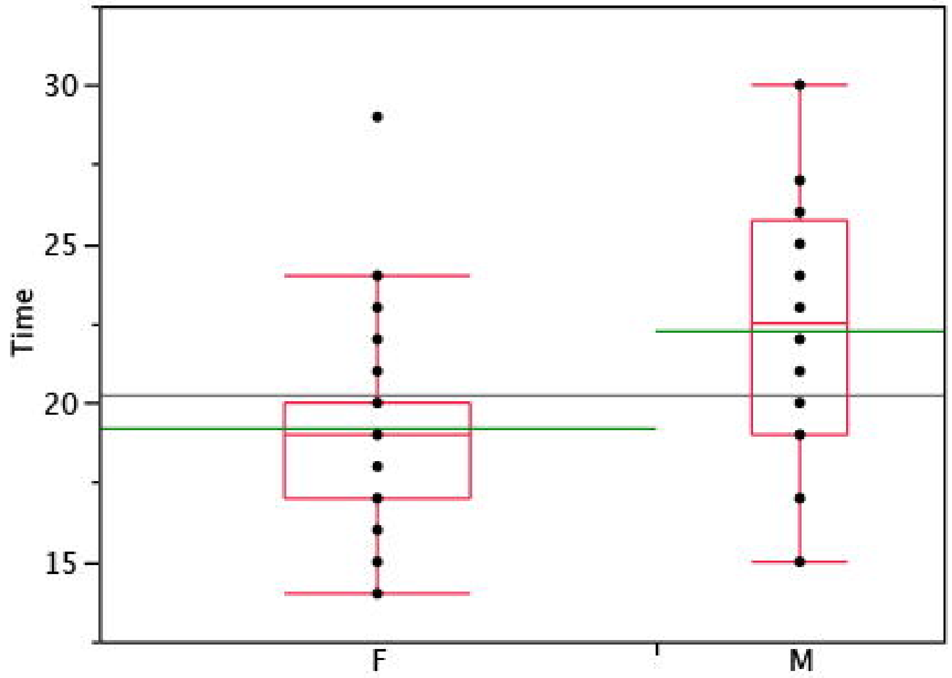
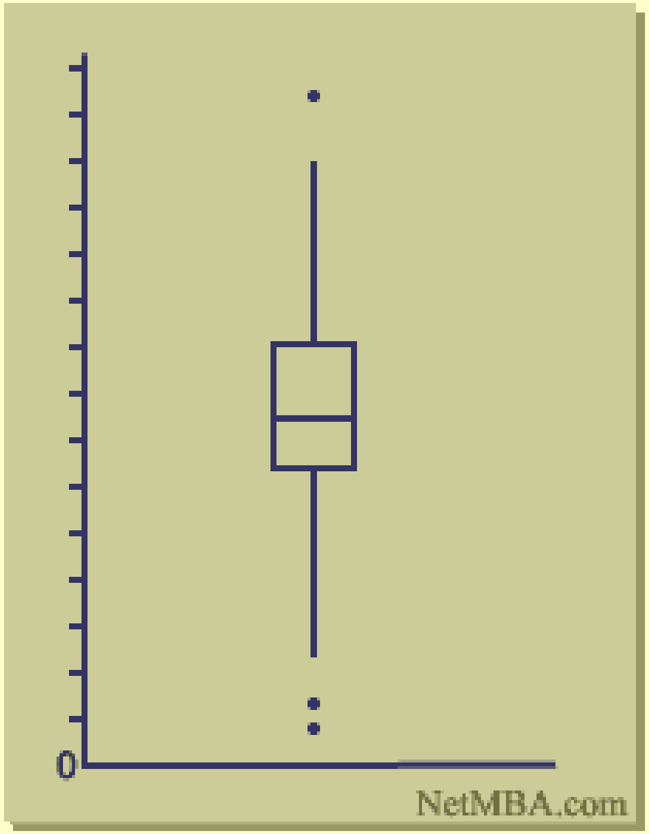


Figure 6. Box plots showing the individual scores and the means.

- http://onlinestatbook.com/2/graphing_distributions/boxplots.html



The plot may be drawn either vertically as in the above diagram, or horizontally.

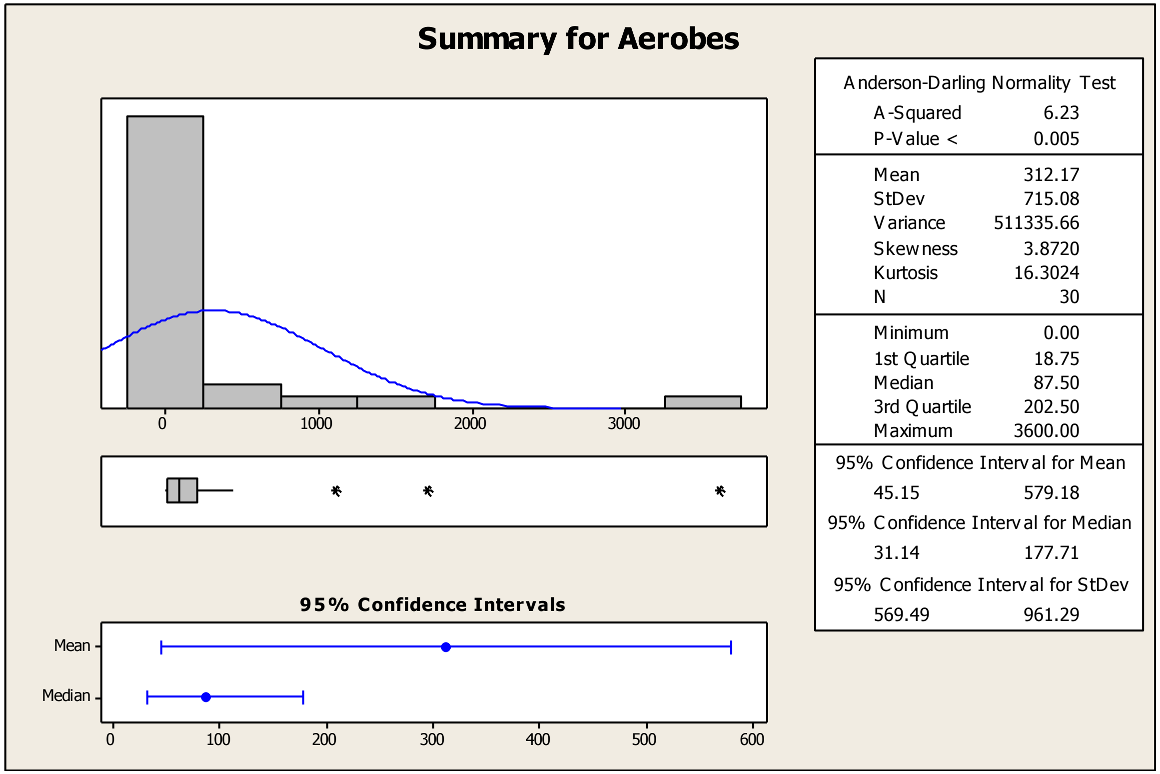
http://www.netmba.com/statistics/plot/box/

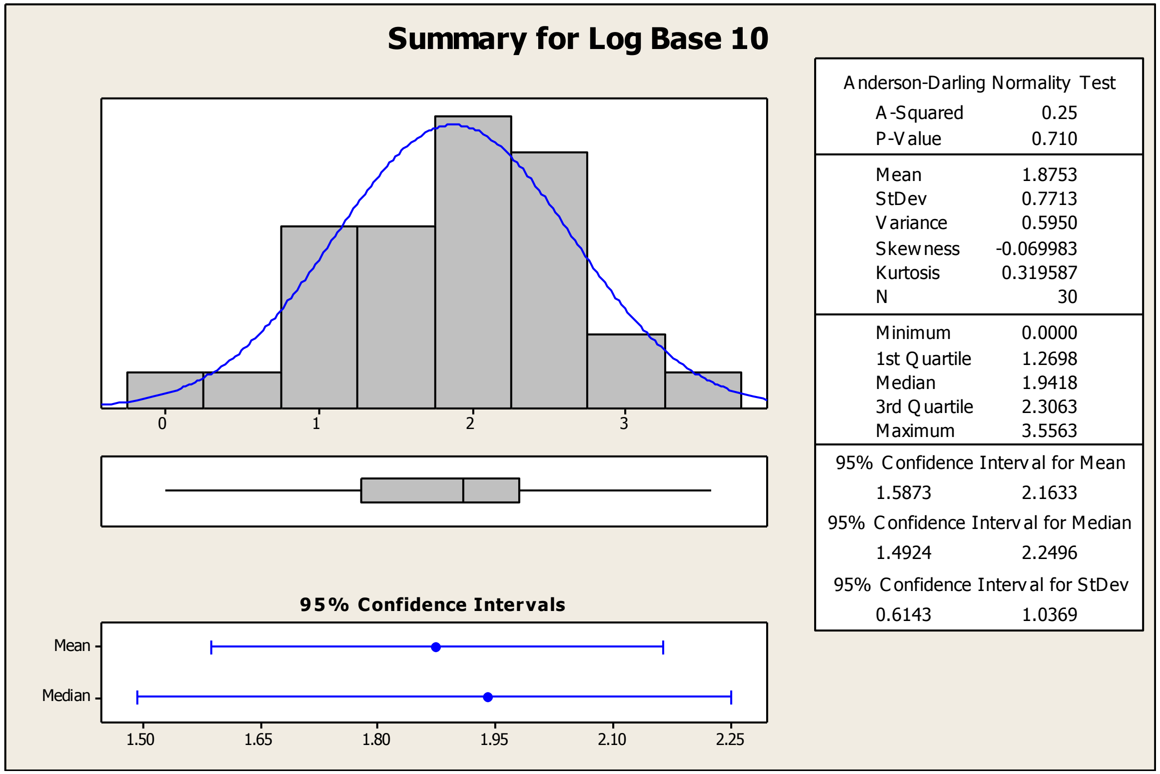
The boxplot is interpreted as follows:

- The box itself contains the middle 50% of the data. The upper edge (hinge) of the box indicates the 75th percentile of the data set, and the lower hinge indicates the 25th percentile. The range of the middle two quartiles is known as the inter-quartile range.
- The line in the box indicates the median value of the data.
- If the median line within the box is not equidistant from the hinges, then the data is skewed.
- The ends of the vertical lines or "whiskers" indicate the minimum and maximum data values, unless outliers are present in which case the whiskers extend to a maximum of 1.5 times the inter-quartile range.
- The points outside the ends of the whiskers are outliers or suspected outliers.

Outlier determination can be tricky!

Data set – 200, 440, 220, 1100, 80, 55, 60, 1700, 100, 200, 160, 110, 170, 210, 3600, 90, 85, 35, 430, 180, 5, 15, 1, 30, 10, 10, 20, 15, 20, 15.





Conclusion

The sample with a reading of 3600 is expected to occur approximately 1 out of every 67 readings on an average and should not be considered extreme based on the distribution of the data. The sample with this count of Aerobes should be identified through a gram stain to determine the nature of its source, but unless abnormal concerns are raised based on the type of microorganism, no additional action should be taken.

- Outliers must be investigated

- Use the tools found in root cause analysis
  - Interview operators
  - Evaluate the measuring system
  - Try to duplicate the reading

- Don’t make assumptions or jump to conclusions

- Do not remove outliers simply because they are outliers

Note that outliers are not necessarily "bad" data-points; indeed they may well be the most important, most information rich, part of the dataset. Under no circumstances should they be automatically removed from the dataset. Outliers may deserve special consideration: they may be the key to the phenomenon under study or the result of human blunders.

Control Charts

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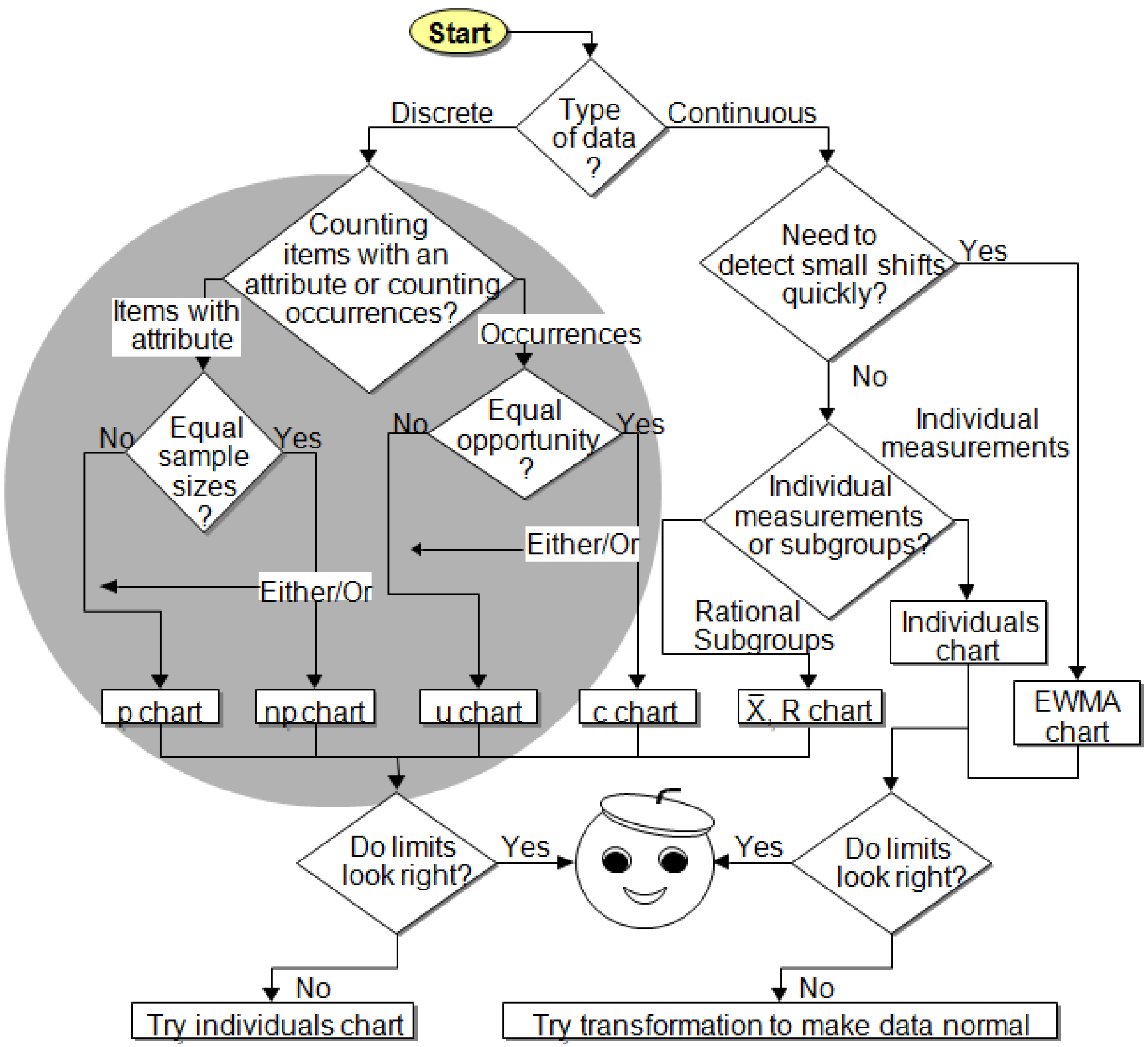
The World of Control Charts



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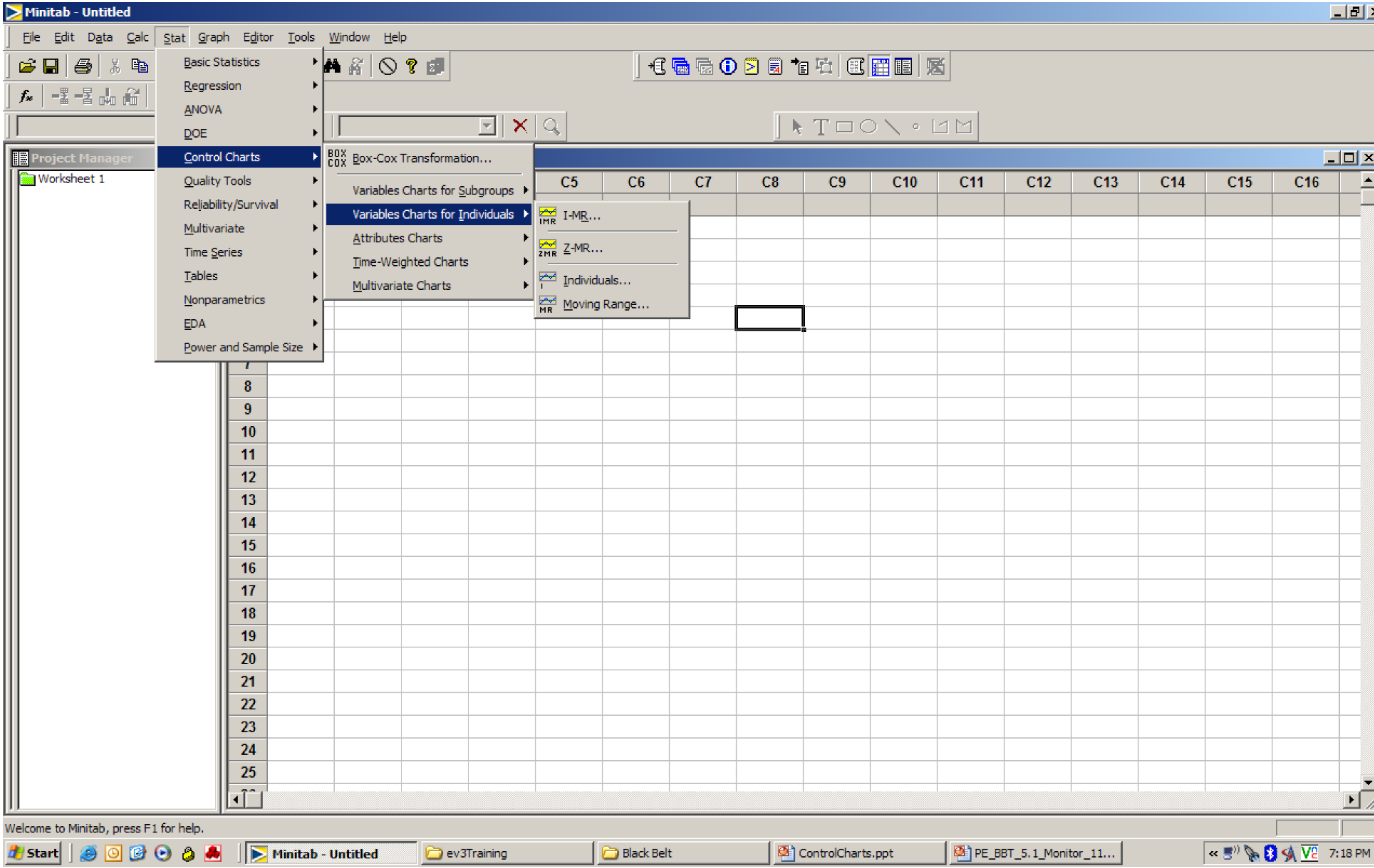
Control Charts



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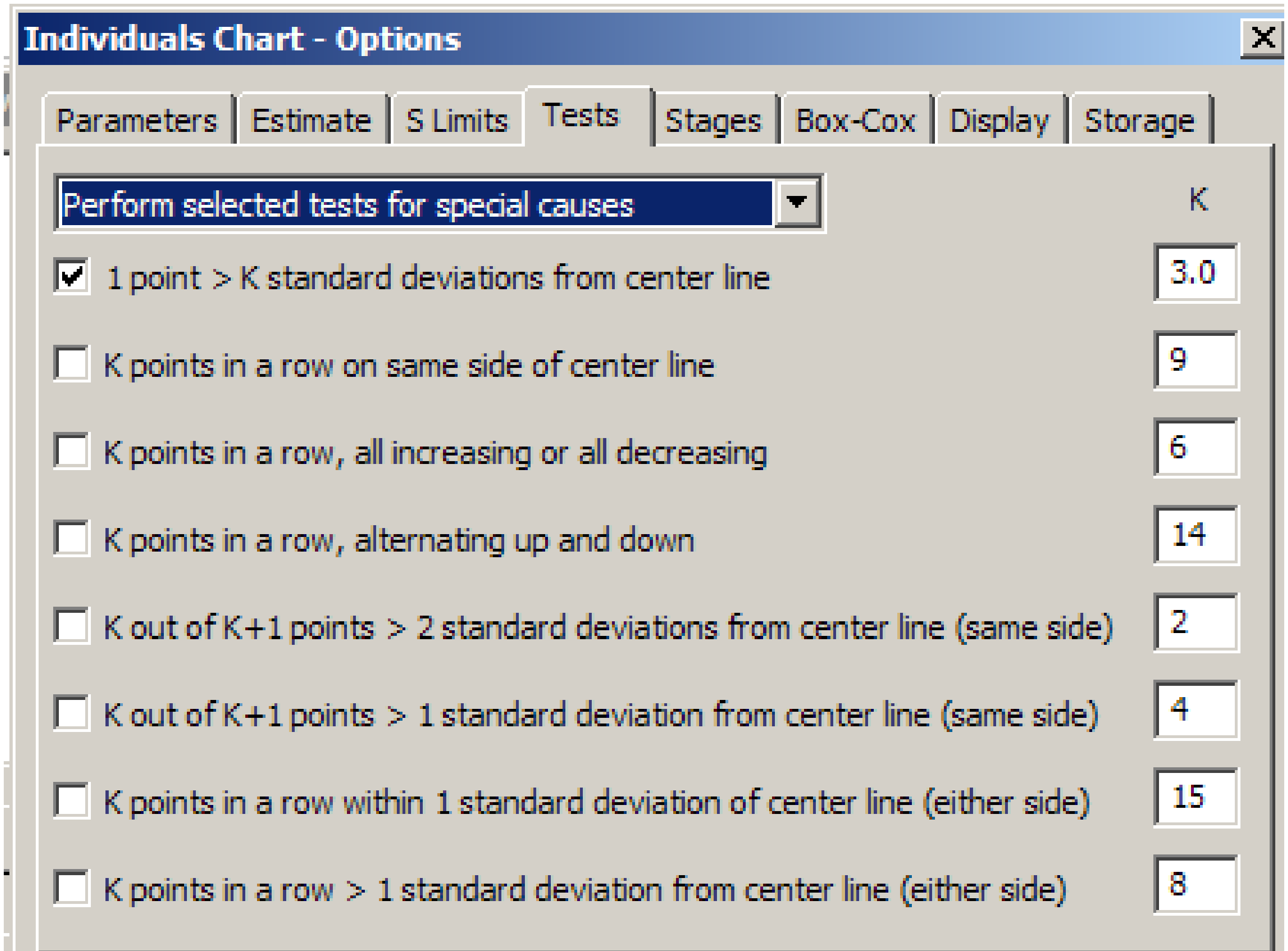
Test to Determine Abnormal Situations (Special Causes)



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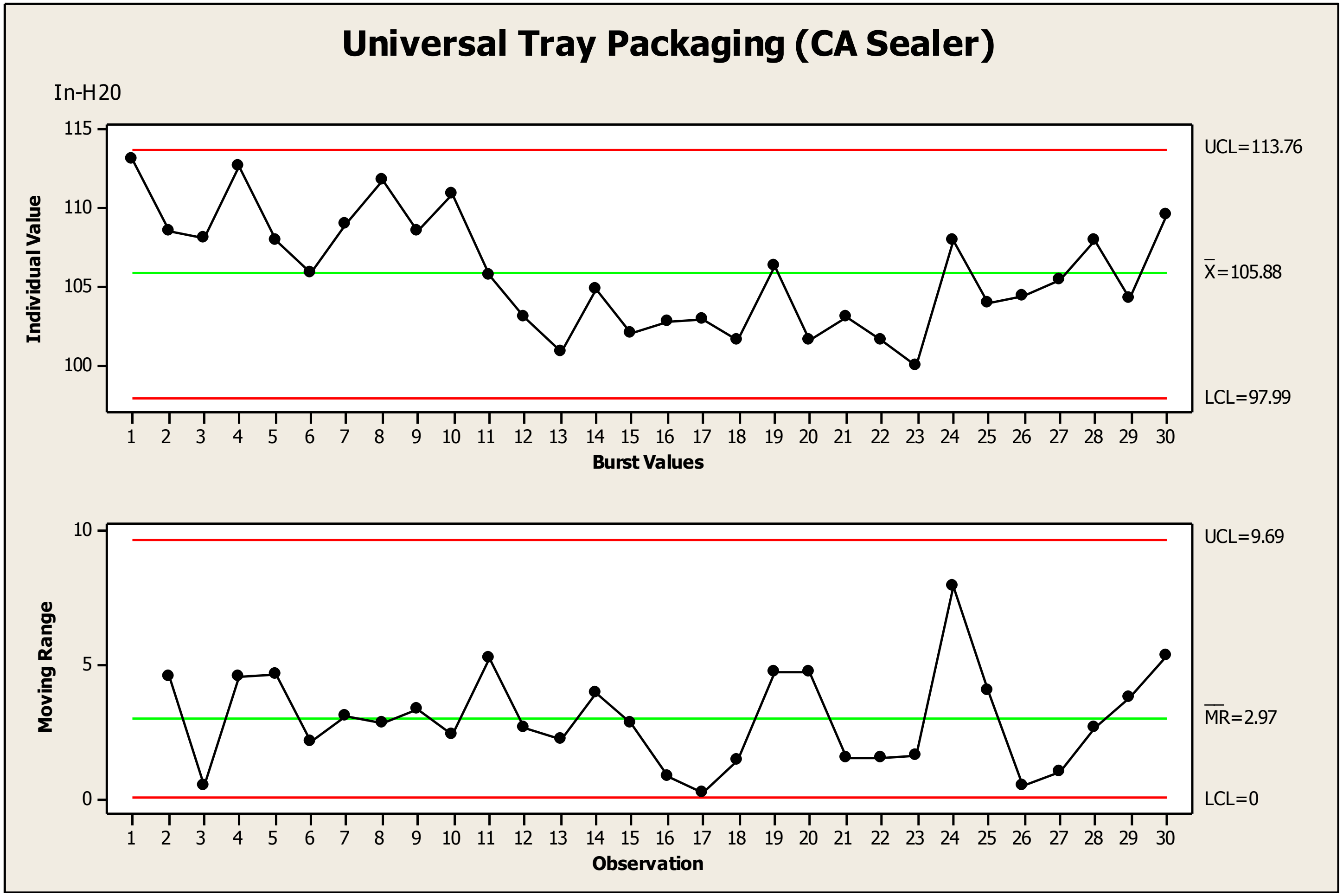
Individual and Moving Range Chart Example



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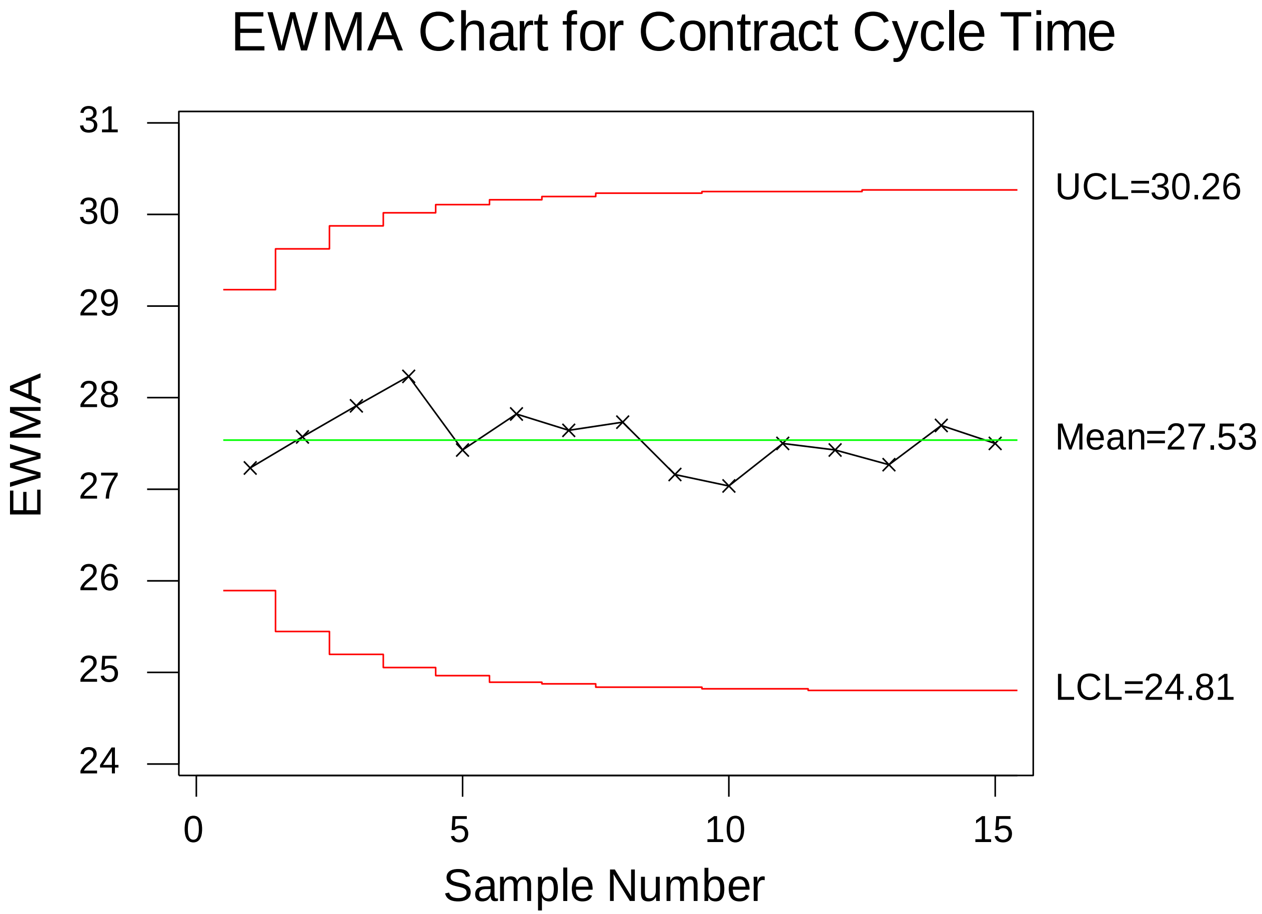
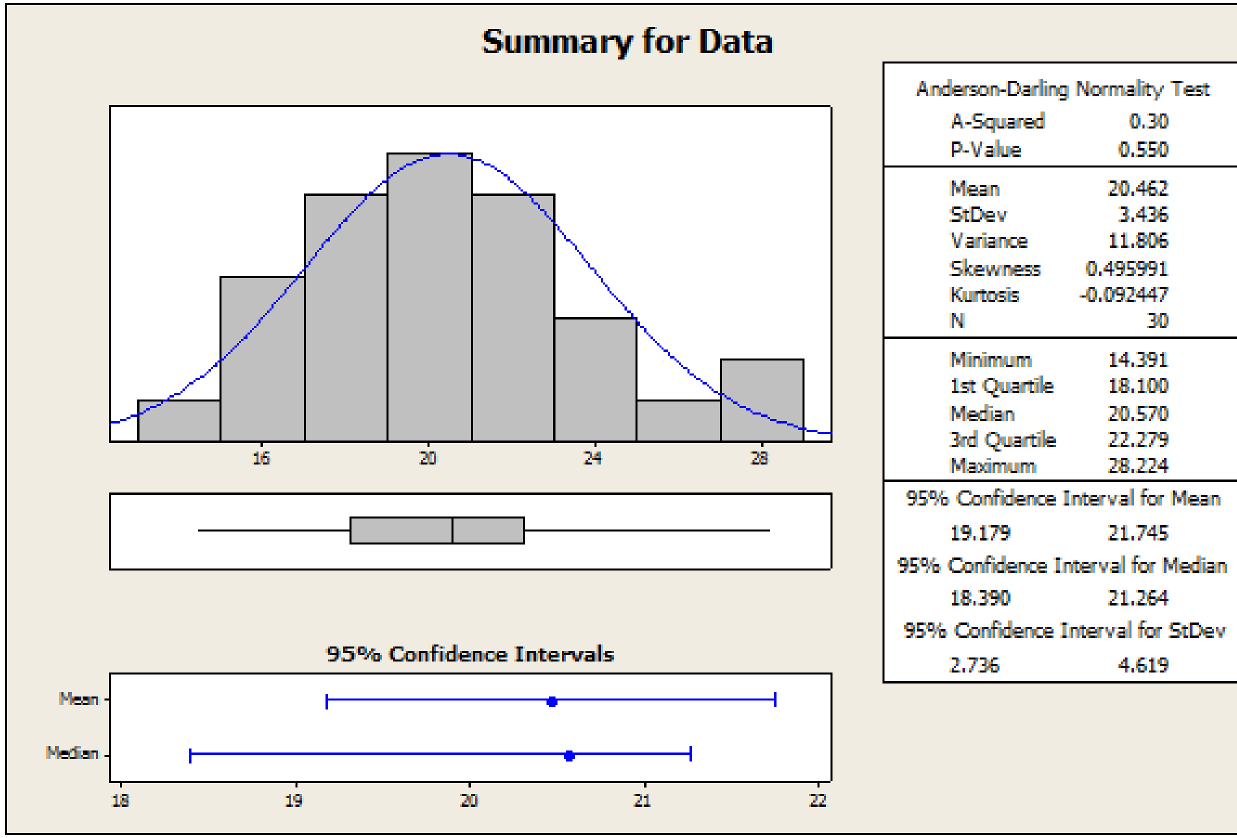
Small Shifts – EWMA Chart

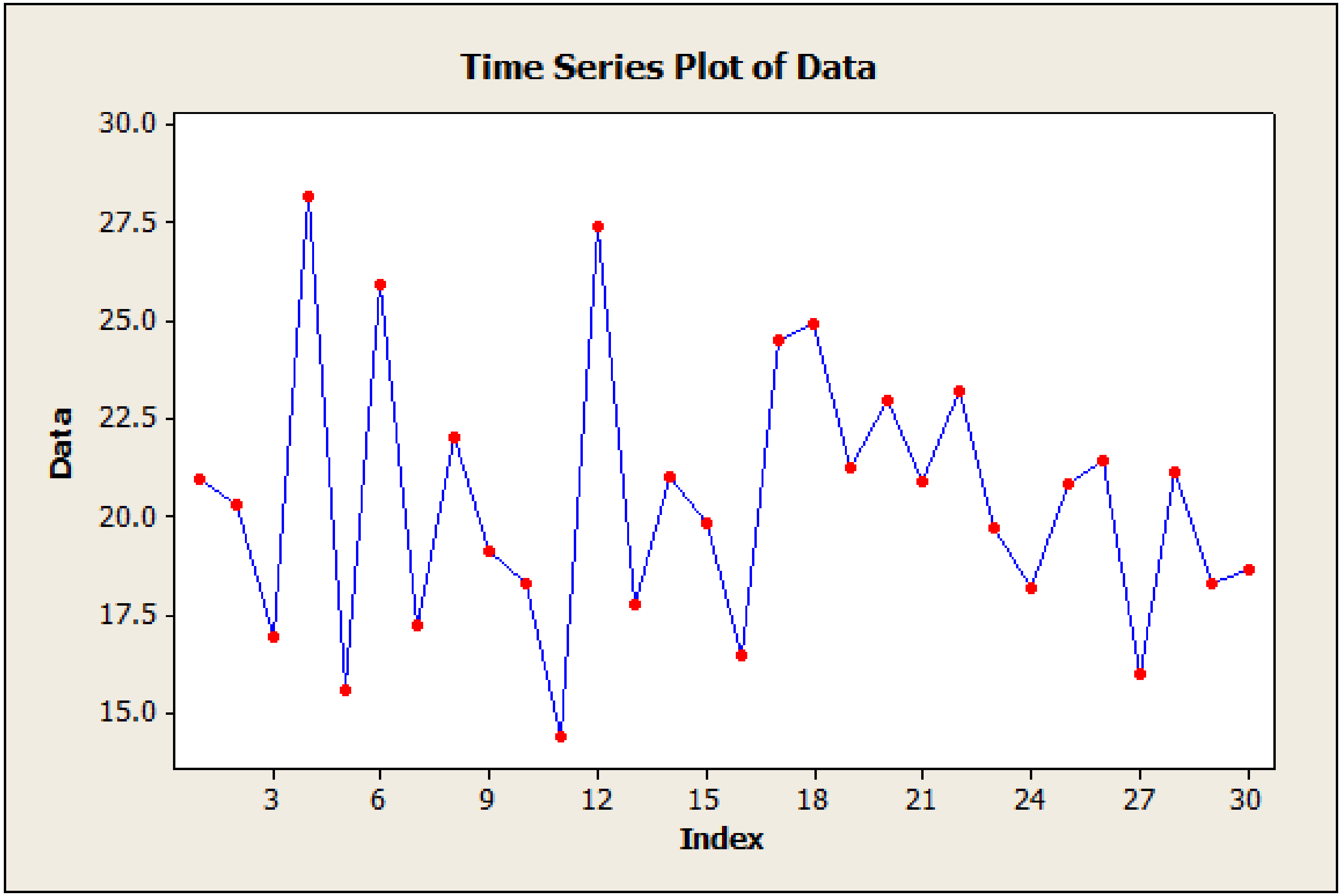


Normal Data for Data Set 1

20.99	16.46
20.32	24.53
16.95	24.95
28.22	21.29
15.61	22.98
25.96	20.91
17.21	23.23
22.05	19.74
19.10	18.21
18.31	20.83
14.39	21.46
27.43	16.00
17.77	21.17
21.02	18.32
19.85	18.63



Time Series Plot for Data 1



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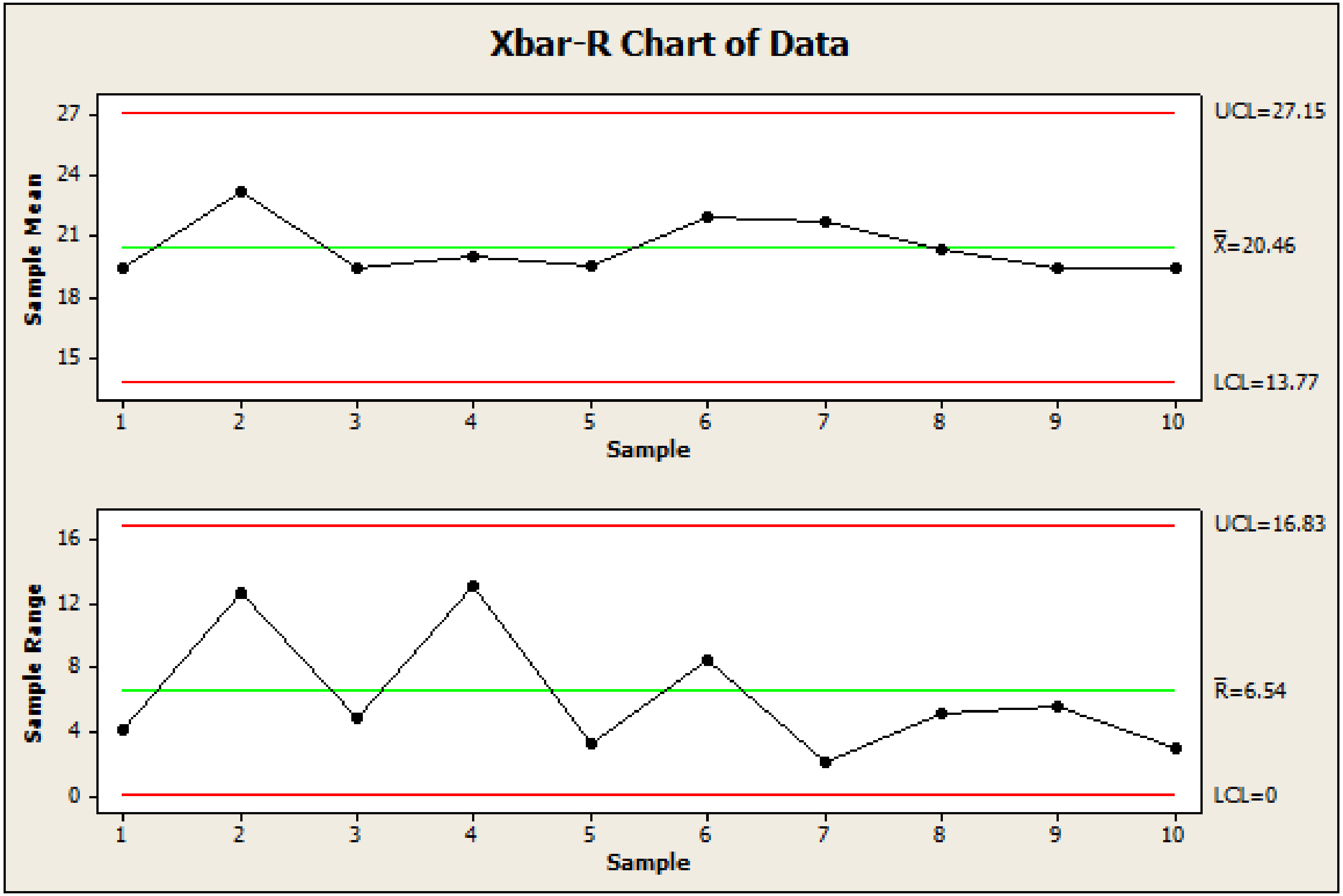
X Bar & R for Data Set 1



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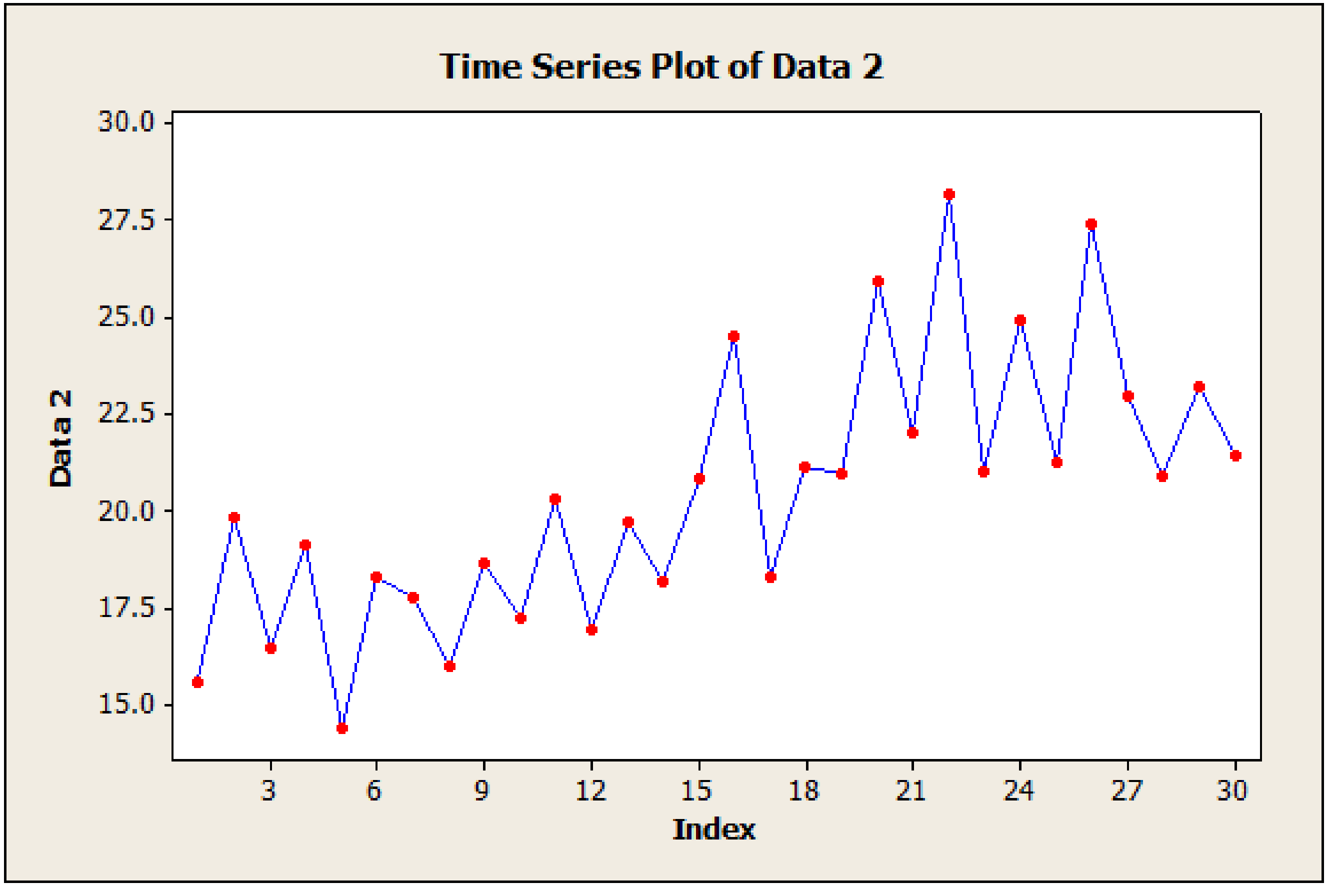
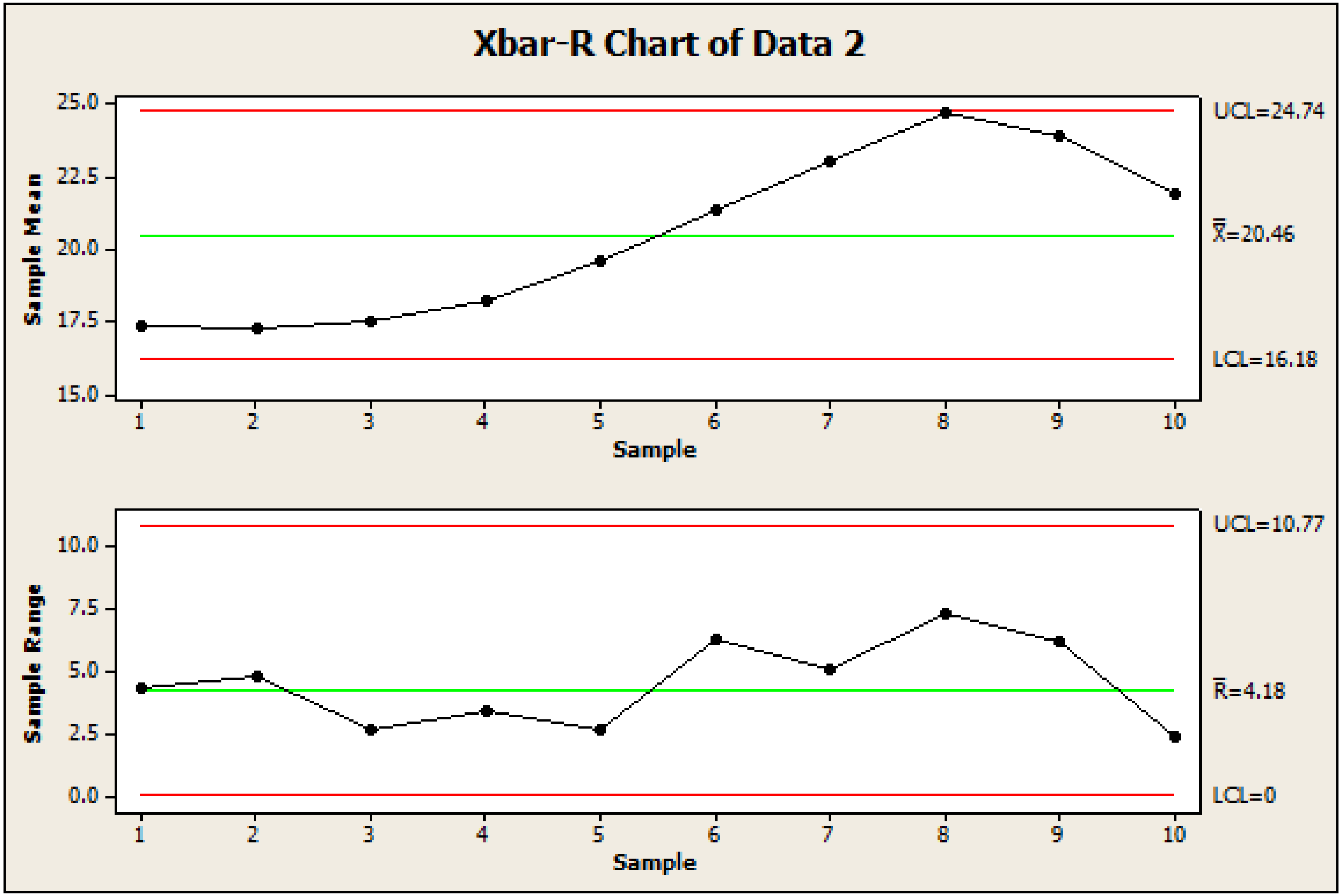
Time Series for Data Set 2

15.61	24.53
19.85	18.32
16.46	21.17
19.10	20.99
14.39	25.96
18.31	22.05
17.77	28.22
16.00	21.02
18.63	24.95
17.21	21.29
20.32	27.43
16.95	22.98
19.74	20.91
18.21	23.23
20.83	21.46



X Bar & R for Data Set 2



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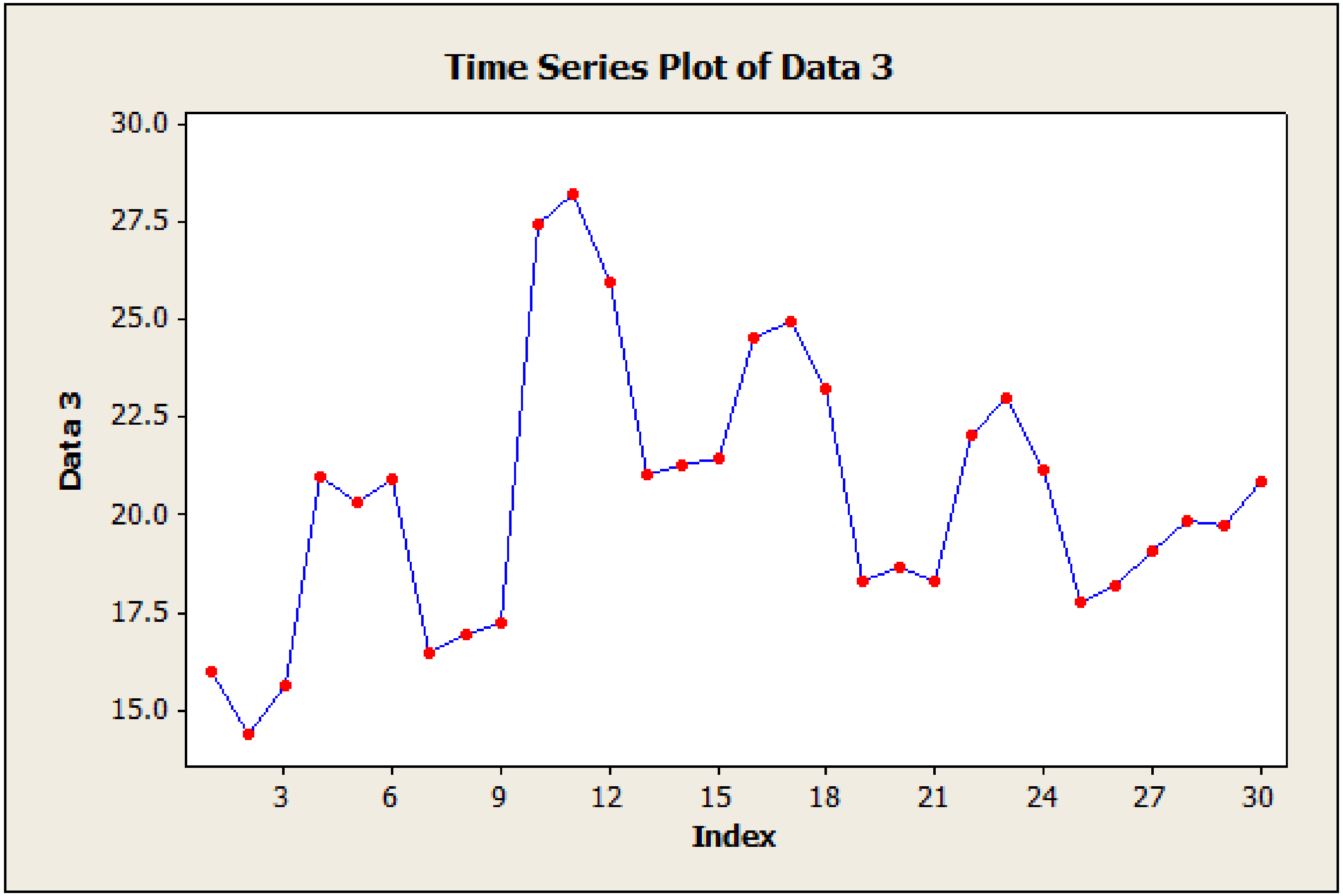
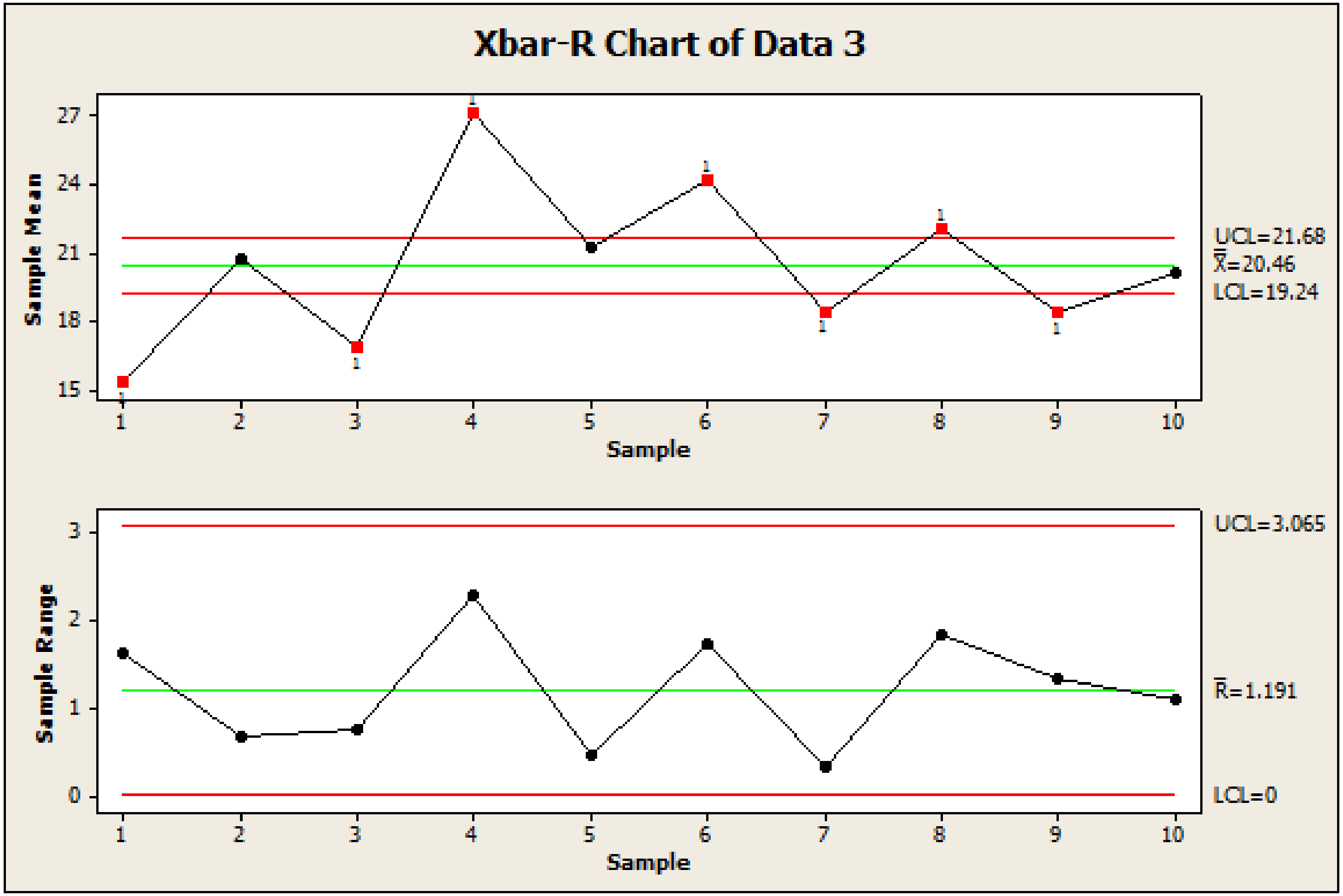
Time Series Plot for Data 3

16.00	24.53
14.39	24.95
15.61	23.23
20.99	18.32
20.32	18.63
20.91	18.31
16.46	22.05
16.95	22.98
17.21	21.17
27.43	17.77
28.22	18.21
25.96	19.10
21.02	19.85
21.29	19.74
21.46	20.83



X Bar & R for Data Set 3



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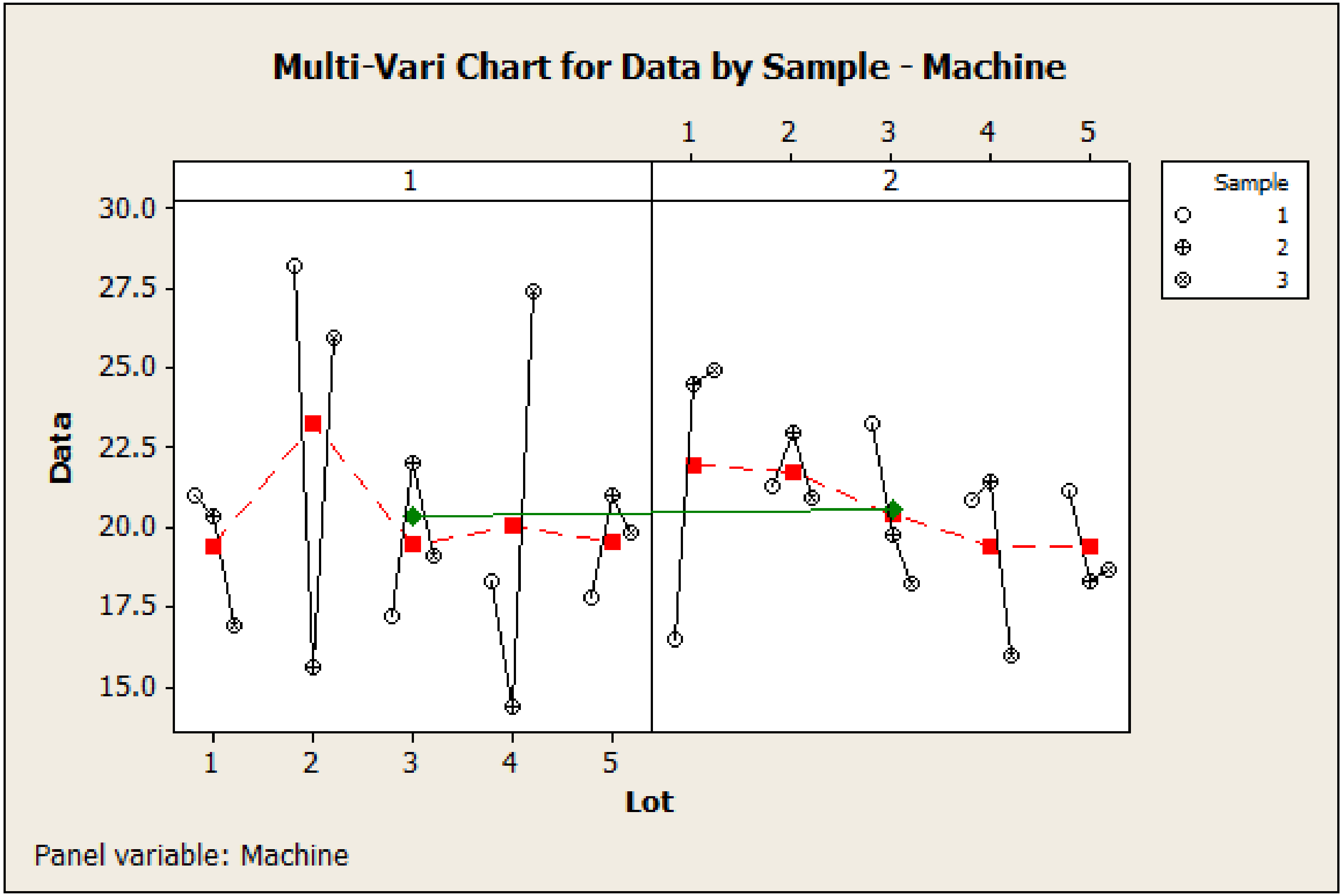
Multi-Vari Chart for Data Set 1



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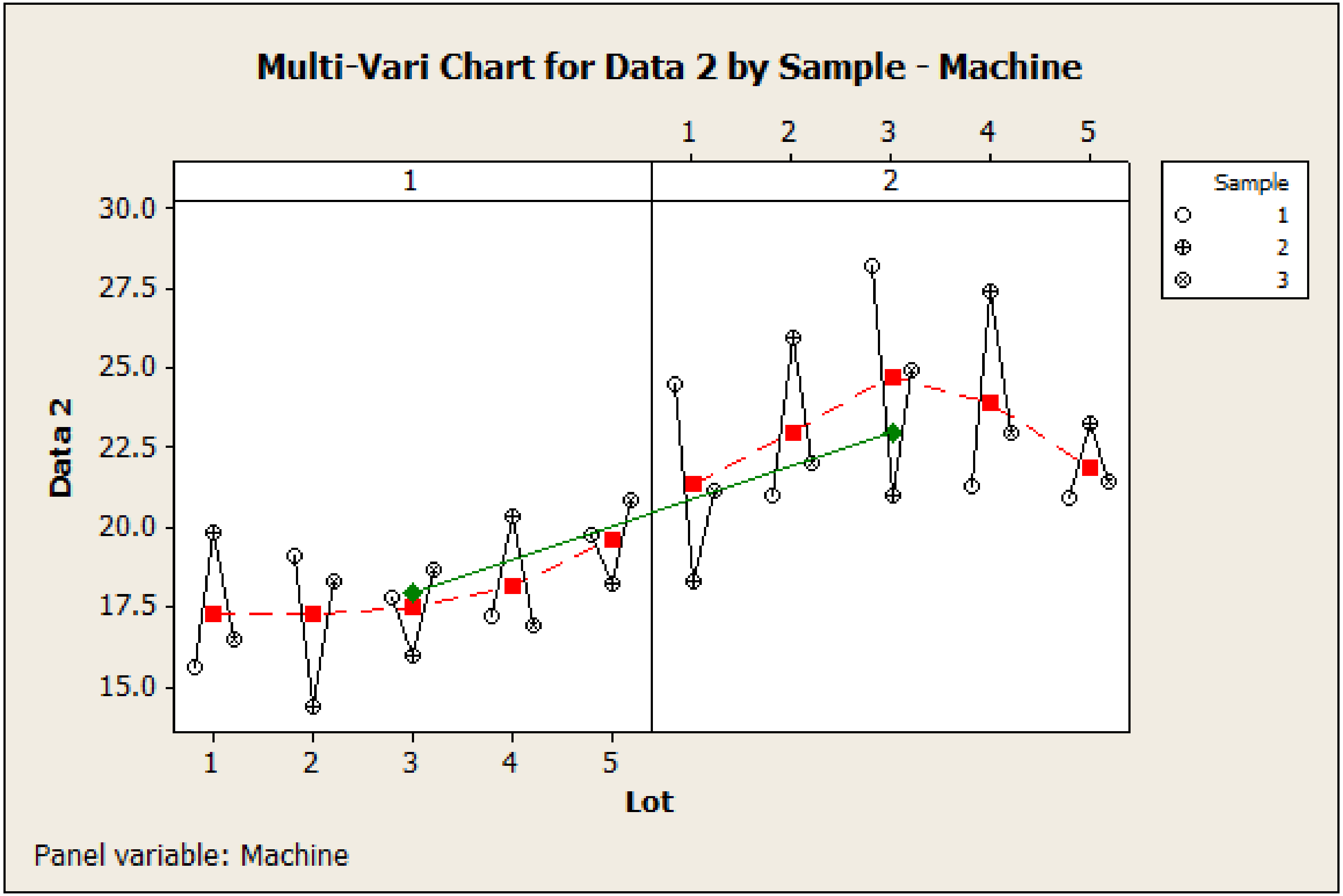
Multi-Vari Chart for Data Set 2



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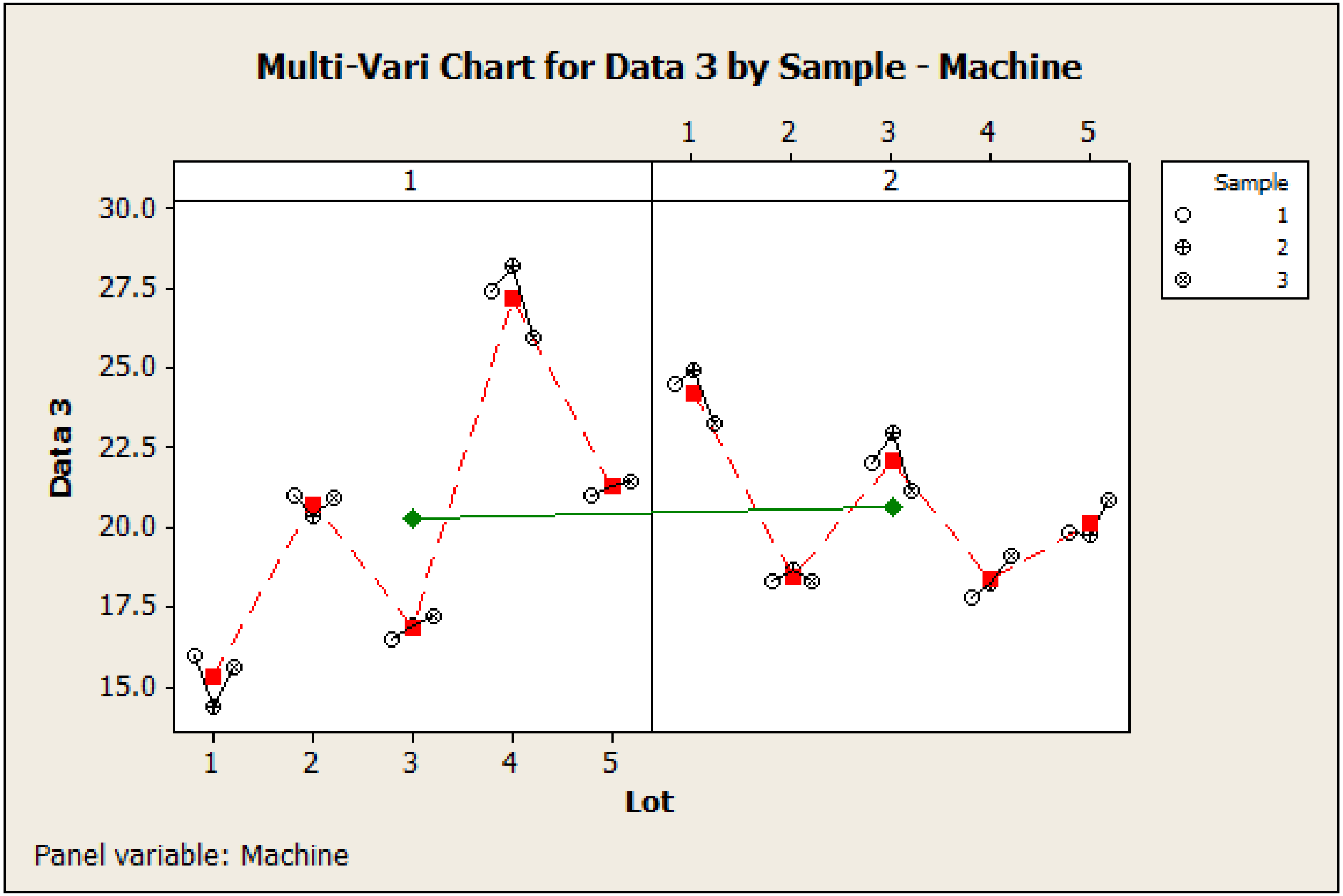
Multi-Vari Chart for Data Set 3



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Questions

Two questions:

1) **What have we learned from the data sets and graphs?**

2) **Will the product resulting from these three different data sets be different in terms of quality or the same?**

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Hypothesis Testing

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Types of Hypothesis Tests

- The first three tests look at differences in group **averages.**
- The 4th looks for differences in **variance.**
- The last 3 look at differences in **proportions.**

Hypothesis Test	**Purpose**
t-test	Compare two group averages
Paired *t*-test	Compare two group averages when data is paired
ANOVA (Analysis of Variance)	Compare two or more group averages
Test of Homogeneity of Variance	Compare two or more group variances
Chi-Square test	Compare two or more group proportions
1-Proportion test	Compare one proportion to a Prescribed boundary
2-Proportion test	Compare two group proportions

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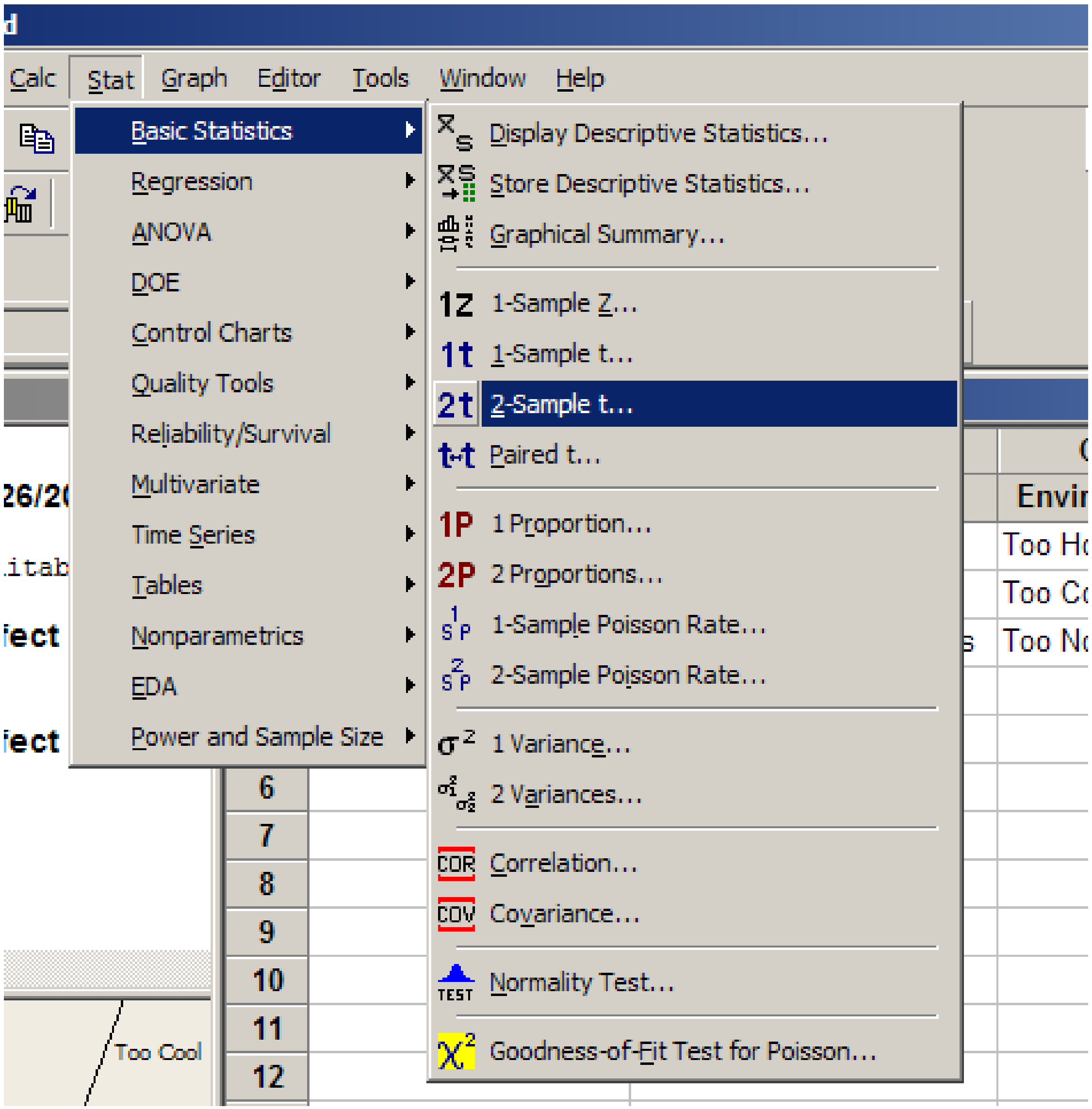
Hypothesis Testing



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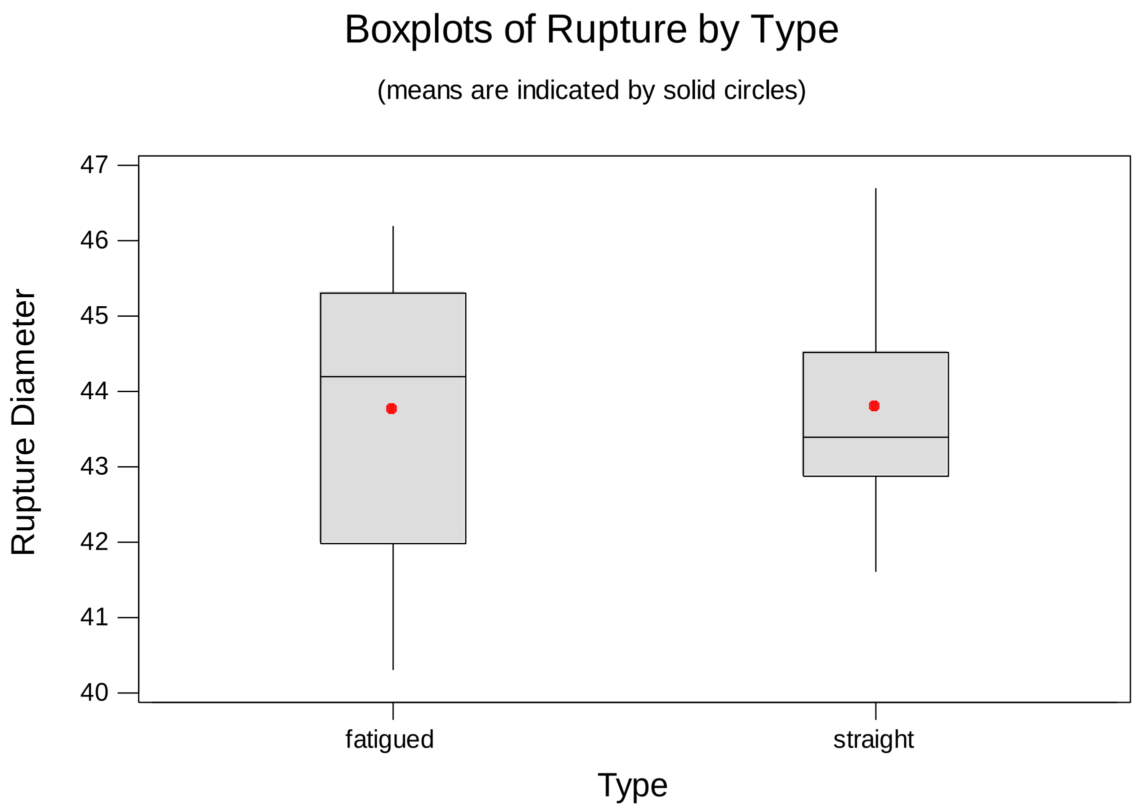
Another Simple Combination

Does fatigue affect mean rupture diameter?



t-test: p=0.96 F-Test p=0.605 Levene's Test p=0.382 ⇒

NO. No statistically significant difference in mean rupture diameter pre and post fatigue…….

The Three t's

- t-tests come in 3 main types
  - <u>One sample t-test</u>
    - Compares a sample to a known value
    - Value can be based on history
  - <u>Two sample t-test</u>
    - Compares two samples
  - <u>Paired t-test</u>
    - Usually compares two treatments to the sample samples
    - Example: two people measuring the same parts

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Let's Talk About P-values

- There are many ways to state the conclusions reached based on a P-value. Which of these are easier to understand?

1. A P-value is used to judge whether an observed difference between groups is significantly bigger than common-cause (random) variation (yes, if $P < .05$).

2. If $P < .05$, reject the H_0 and conclude the H_a.

3. $P < .05$ means that there is less than a 5% chance that the groups came from the same distribution.

4. A P-value determines whether the observed difference is a statistically significant difference (yes, if $P < .05$).

5. The P-value equals the probability of obtaining the observed difference given that the "true" difference is zero.

6. If the P-value is low ($< .05$), the observed difference must be significant since the probability is low that such a difference in samples could be observed, if indeed there was no "true" difference.

7. A P-value is used to judge whether there is enough statistical evidence to reject the null hypothesis (yes, if $P < .05$).

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One-sample t-test

The one-sample t-test assumes the population is normally distributed. However, it is fairly robust to violations of this assumption, provided the observations are collected randomly and the data are continuous, unimodal, and reasonably symmetric.

Why use a one-sample t-test?
A one-sample t-test can help answer questions such as:

- **Is the process on target?**
- **Does a key characteristic of a supplier's material have the desired mean value?**
- **Is a new material or process taking us in the right direction?**

Let's discuss this last one. Are there times I want to reject the Null Hypothesis?

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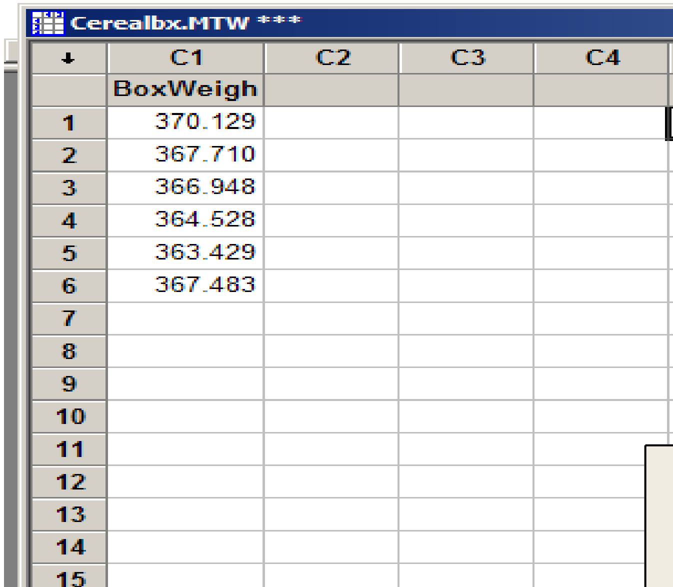
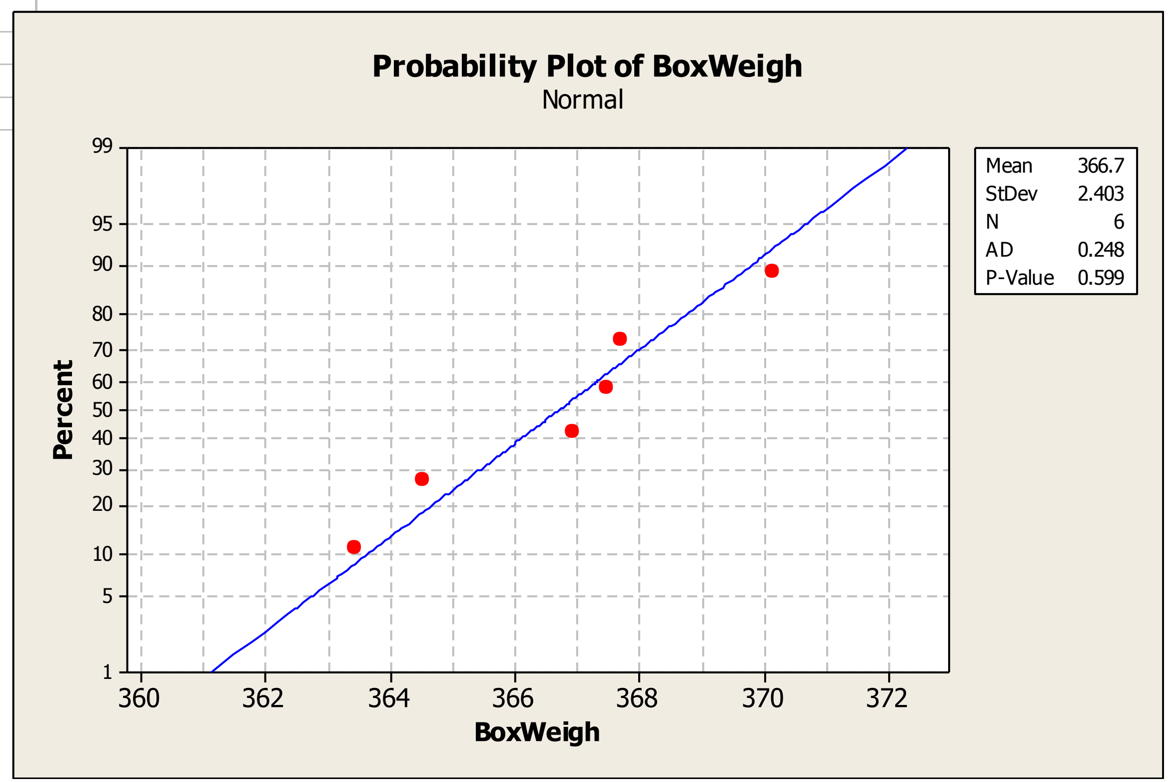
One-sample t-test example



Cerealbx.MTW ***

↓	C1	C2	C3	C4
	BoxWeigh			
1	370.129			
2	367.710			
3	366.948			
4	364.528			
5	363.429			
6	367.483			
7				
8				
9				
10				
11				
12				
13				
14				
15				

We want to determine if the box weight is at our target of 365 grams.



We need to start with a check of the normality assumption.

What is the hypothesis here?

The Analysis

```
Welcome to Minitab, press F1 for help.
Retrieving project from file: 'D:\MANUFACTURING\BASIC STATISTICS
1.0\CEREALBX.MPJ'
```

Results for: Cerealbx.MTW

Probability Plot of BoxWeigh

Probability Plot of BoxWeigh

One-Sample T: BoxWeigh

```
Test of mu = 365 vs not = 365


Variable  N     Mean   StDev  SE Mean         95% CI               T      P
BoxWeigh  6  366.705   2.403    0.981  (364.183, 369.226)  1.74  0.143
```

The Paired t-test

The paired t-test uses matched data

Matched data can be for analysis of:

- Two Operators
- Two Materials
- Two Suppliers
- Two Machines
- Two Pieces of Measuring Equipment

It is statistically more stringent than a two-sample t-test

Let's try to understand why.

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An Exercise with the t test

SCRAP.MTW ***

↓	C1	C2	C3	C4
	Cell	New$Scrap	Std$Scrap	
1	1	101.97	114.71	
2	2	126.17	134.40	
3	3	133.63	133.68	
4	4	173.98	185.04	
5	5	206.26	209.60	
6	6	161.11	168.23	
7	7	99.63	109.02	
8	8	133.82	131.06	
9	9	154.88	161.46	
10	10	141.72	146.78	
11				

Let's do treat this data as paired and also as unpaired to see if there is a difference.

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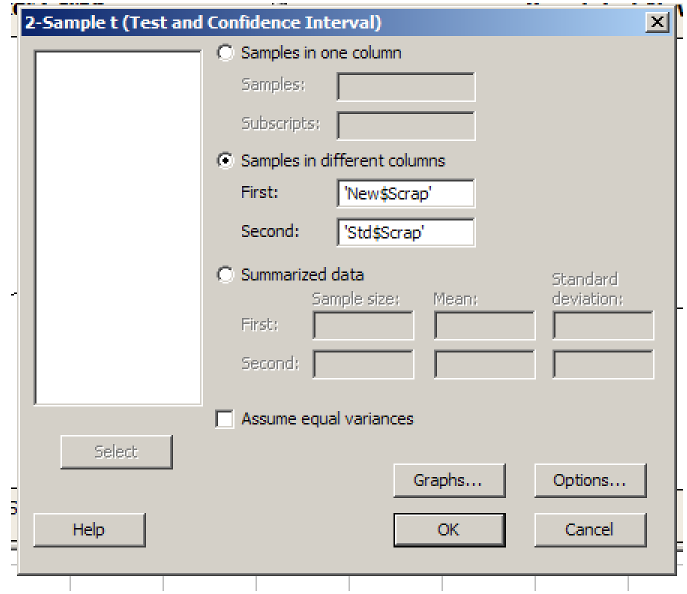
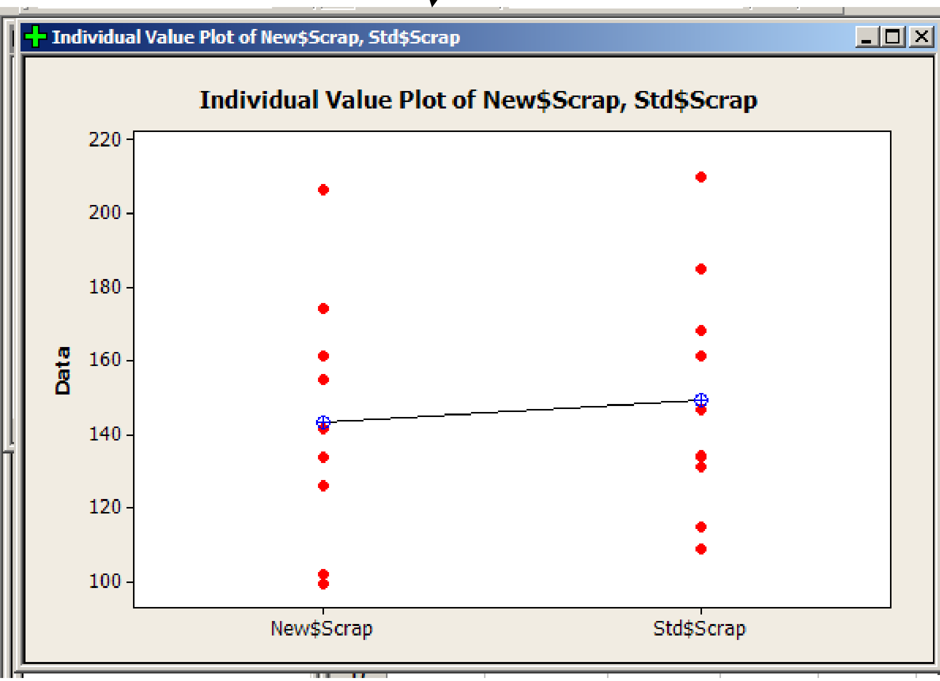
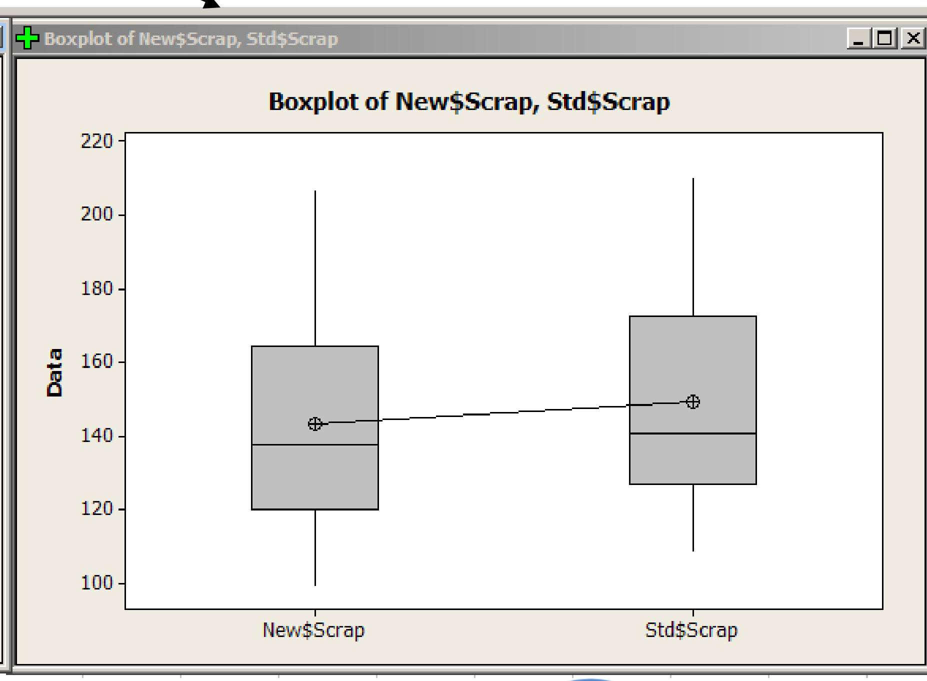


Here's The Graphs of the unpaired approach





And the session windows

2-sample t

```
Difference = mu (New$Scrap) - mu (Std$Scrap)
Estimate for difference:  -6.1
95% CI for difference:  (-36.3, 24.2)
T-Test of difference = 0 (vs not =): T-Value = -0.42  P-Value = 0.677  DF = 17
```

Paired t

```
Paired T for New$Scrap - Std$Scrap

             N   Mean  StDev  SE Mean
New$Scrap   10  143.3   32.4     10.2
Std$Scrap   10  149.4   31.7     10.0
Difference  10  -6.08   4.82     1.52


95% CI for mean difference: (-9.53, -2.63)
T-Test of mean difference = 0 (vs not = 0): T-Value = -3.99  P-Value = 0.003
```

What 2 Key indicators are there regarding the Null Hypothesis?

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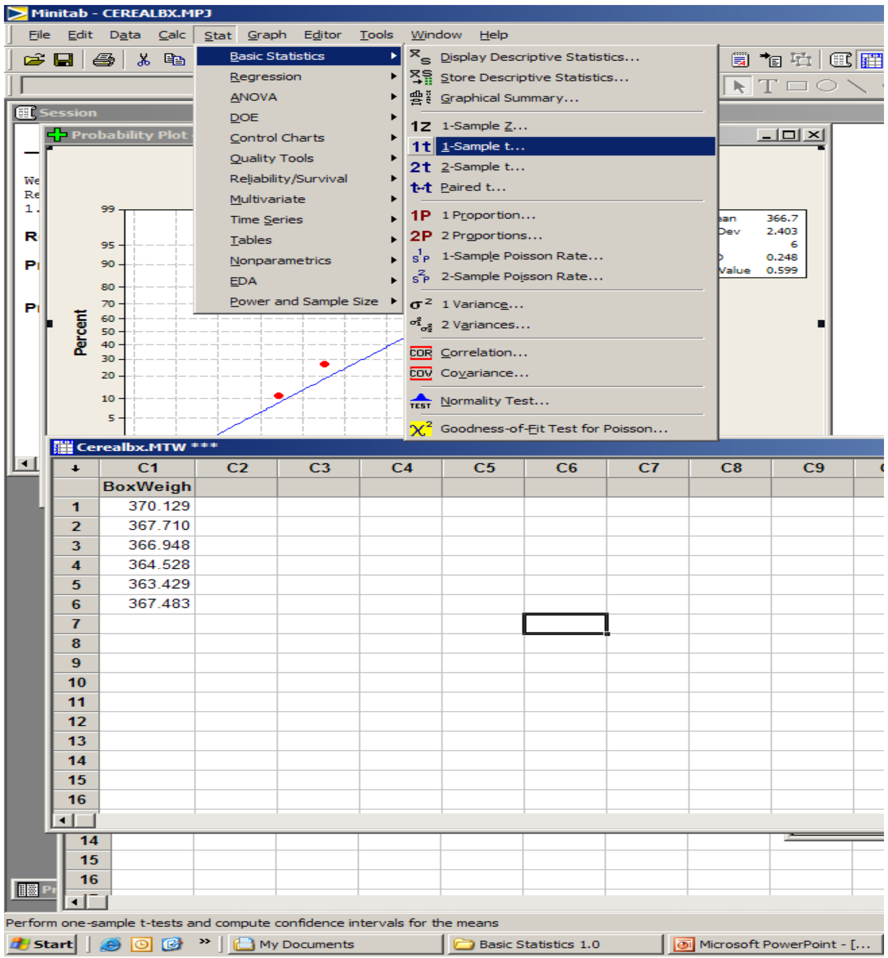
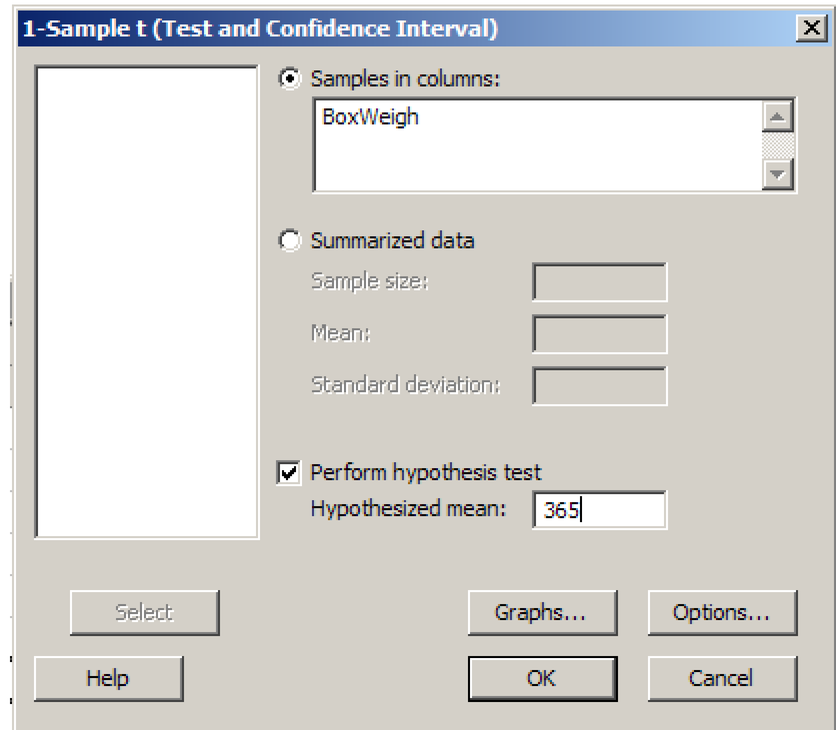
But Why?

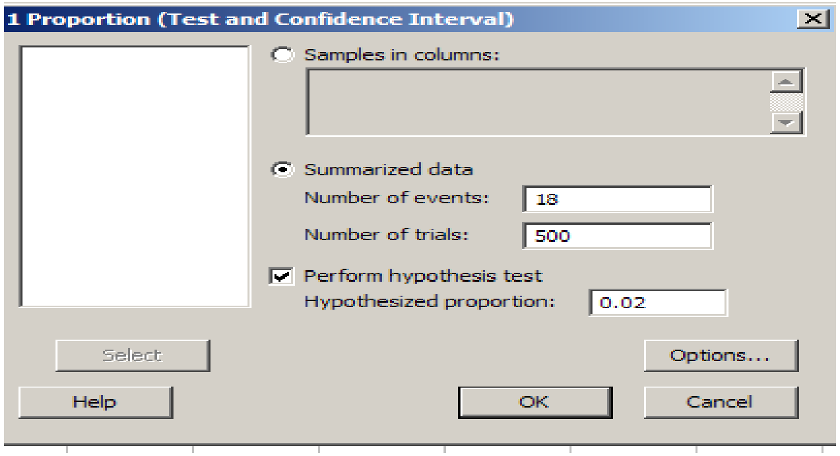
Cell	New $Scrap	Standard $Scrap	Paired Differences
1	101.97	114.71	12.74
2	126.17	134.40	8.23
3	133.63	133.68	0.05
4	173.98	185.04	11.06
5	206.26	209.60	3.34
6	161.11	168.23	7.12
7	99.63	109.02	9.39
8	133.82	131.06	-2.76
9	154.88	161.46	6.58
10	141.72	146.78	5.06
average	143.32	149.40	6.08
st.dev.	32.41	31.70	4.82

Executing the test

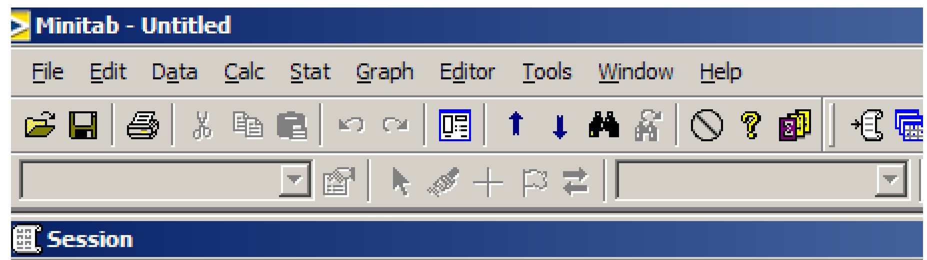




One-portion test



We'll use summarized data for this one



4/29/2009 9:35:25 AM

```
Welcome to Minitab, press F1 for help.
```

Test and CI for One Proportion

```
Test of p = 0.02 vs p not = 0.02


                                                            Exact
Sample   X    N  Sample p         95% CI             P-Value
1       18  500  0.036000  (0.021473, 0.056300)        0.023
```

What statements can we now make about the process compared to a target of 2%?

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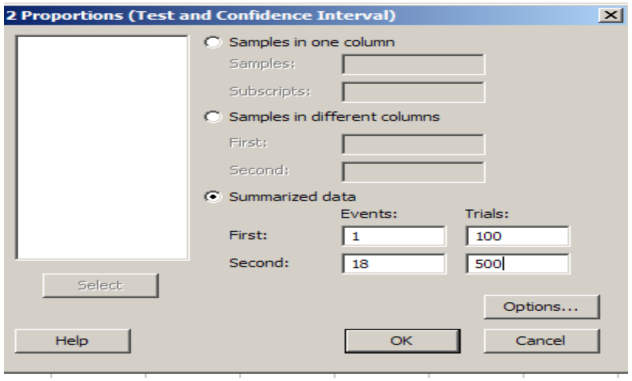
Two-portion test



We can compare two samples this way.

```
1       18  500  0.036000  (0.021473, 0.056300)    0.023
```

Test and CI for Two Proportions

```
Sample    X    N  Sample p
1         1  100  0.010000
2        18  500  0.036000


Difference = p (1) - p (2)
Estimate for difference:  -0.026
95% CI for difference:  (-0.0514349, -0.000565134)
Test for difference = 0 (vs not = 0):  Z = -2.00  P-Value = 0.045

Fisher's exact test: P-Value = 0.225


* NOTE * The normal approximation may be inaccurate for small samples.
```

What statements can we make about these two samples?

Process Capability

Capability Studies

1) The true purpose of a capability study is to determine exactly how well a process, product, component can meet a predetermined specification.
2) It can be measured in several different ways.
3) The values generated can be absolute or relative. We may want to compare or we may want to use it to check against a hard and fast standard.

A Capability Study consists of:

Measuring Devices, Procedures, Definitions, People, Specifications and Processes.

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7 Steps to a Good Process Capability Study

1. Obtain Process Data

2. Prepare Control Charts and Frequency Displays

3. Evaluate Process Stability

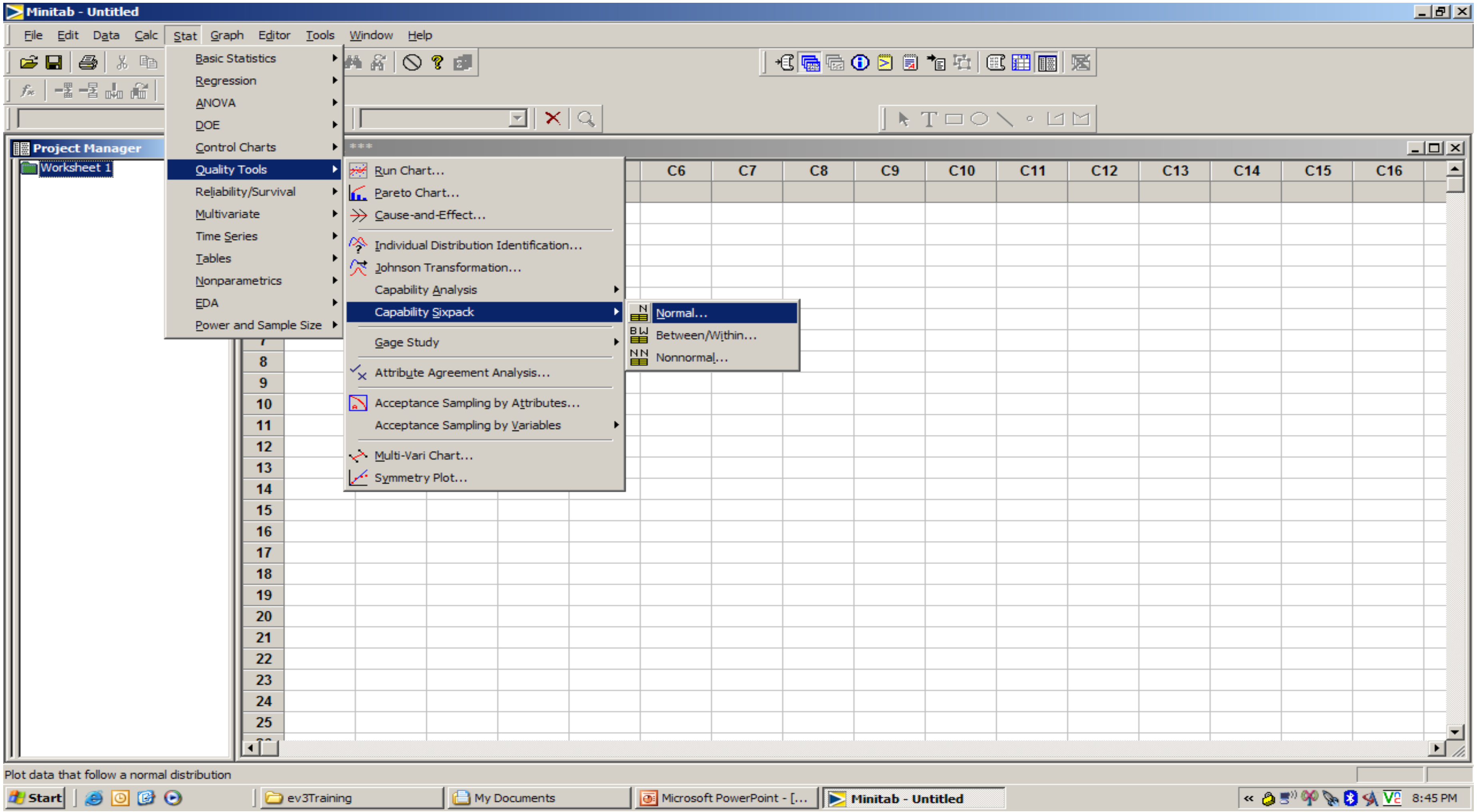
4. Estimate Short-Term and Long-Term Process Variation

5. Evaluate how well a Normal Distribution approximates the individual process observations

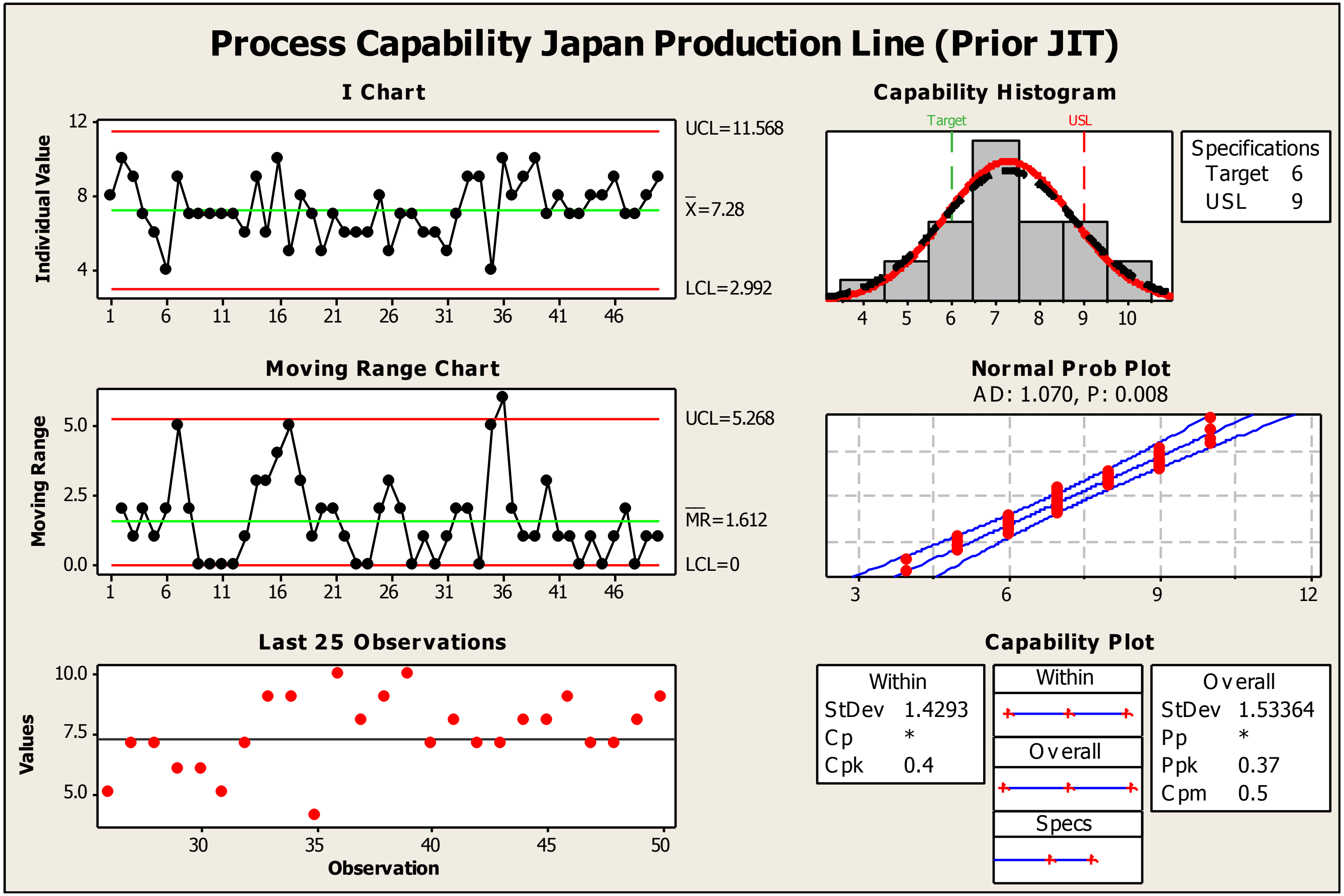
6. Prepare Summary Measures of Process Capability

7. Judge adequacy of Process Capability

Where is it in Minitab ?



Process Capability Example



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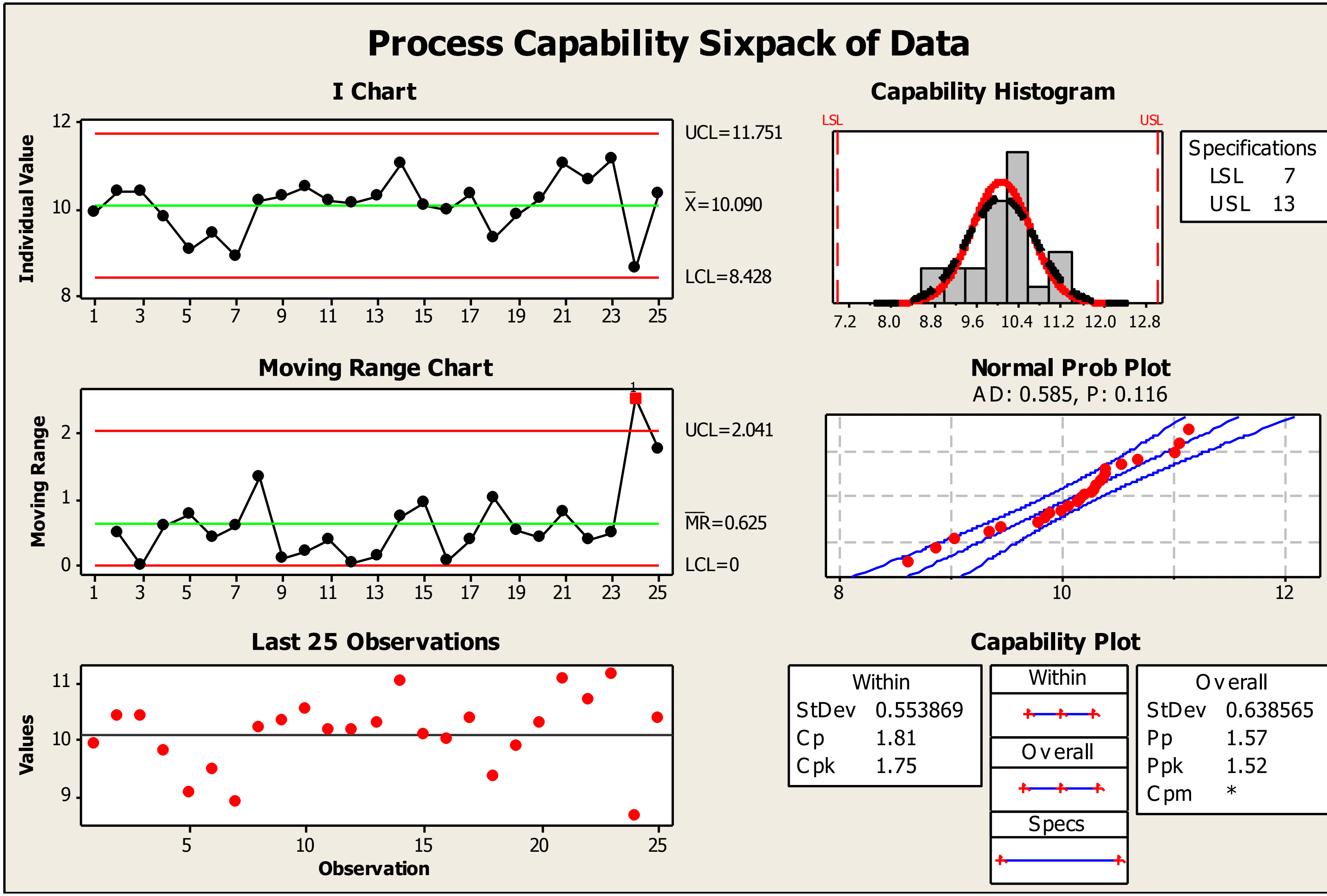

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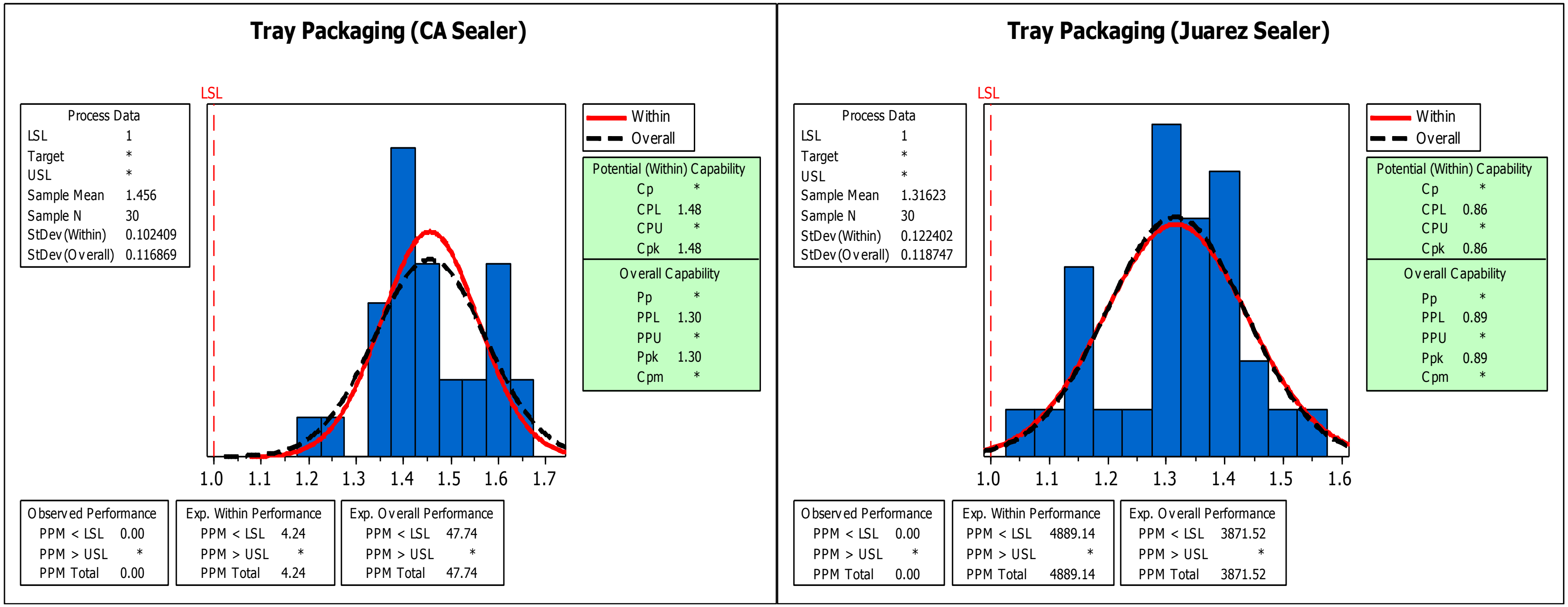
How About Using Process Capability for Comparative Purposes?



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Thank You For Your Time and Attention !

Uncle Larry